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THE *Country-Survey-Book:* OR, LAND-METERS *VADE-MECUM.*

WHEREIN

The PRINCIPLES and *Practical* RULES for *Surveying of Land*, are so plainly (though briefly) delivered, that any one of ordinary parts (understanding how to *add*, *subtract*, *multiply* and *divide*,) may by the help of this small *Treatise* alone, and a few cheap *Instruments* easy to be procured, *Measure* a parcel of Land, and with judgment and expedition *Plot* it, and give up the Content thereof.

WITH

An *APPENDIX*, containing *Twelve Problems* touching *Compound Interest* and *Annuities*; and a *Method* to *Contract* the work of *Fellowship* and *Alligation Alternate*, very considerable in many Cases.

Illustrated with Copper Plates.

By *ADAM MARTINDALE*,
A Friend to Mathematical Learning.

Frustra fit per plura quod fieri potest per pauciora.

London, Printed for *R. Clavel*, at the *Peacock* in St *Paul's* Chuch-yard, and *G. Sawbridge*, at the *Three Flower-de-luces* in *Little-Britain*, 1702.

The PREFACE.

READER,

THe Title ſufficiently informs thee in general of my Deſigns: But I confeſs I owe thee a a more particular Account, not only of that, but ſeveral other things which I ſhall briefly give thee.

1. I have obſerved that the Country aboundeth with ſuch, as by their Inclination and Intereſt are prevailed with to take pains in meaſuring Land, that for want of better Inſtruction uſe ill divided Chains and tedious Methods of Computation, which makes their work intolerable troubleſom, if exactnaſs be required. And ſome for want of skill in the Fundamentals of Geometry, have imbibed prodigiouſly falſe Principles, as this for one *viz.* That *the Content of any Cloſe, of what Figure ſoever, may be found by ſquaring a quarter of the Perimeter.* Mathematical Schools, where better things might be learned, are very rare, and an able Artiſt to inſtruct one in private is hard, and chargeable to be procured. Excellent Books indeed there are in our *Engliſh* Tongue, Written by our Famous *Rathborn*, *Wing*, *Leybourn* and *Holwell*, to which may be added Induſtrious Mr *Atwel*'s Treatiſe, and ſome part of Capt *Sturmy*'s: But thoſe I rather eſteem fit to be read by an able Artiſt (towards his perfecting) than by a new beginner, for in the beſt of thoſe Books he will find the moſt uſeful and plain Rules ſo intermixed

with others that are leſs neceſſary, and more intricate, (though very excellent for their proper ends) and ſo many Curioſities touching Trigonometry, Tranſmutation of Figures, *&c.* which his buſineſs never calls for, that for want of judgment to pick out that which fits his preſent purpoſe, and to ſtudy higher Speculations afterwards, he is apt to be confounded and diſcouraged; whereby it accidentally comes to paſs that plenty makes him poor.

Beſides, three of theſe ſix Books are in Folio, another in Quarto, and the other two (though Octavo's) too large for ordinary carrying in a Pocket, and in that regard not ſo convenient for one that hath much occaſion to be out of his own Houſe; to ſay nothing of their price, which to ſome poor Youths is not the leaſt diſcouragement.

I have therefore made my Book ſo little, that the price can neither much empty the Pocket, nor the Bulk over-fill it. And yet ſo plain, that I doubt not to be underſtood by very ordinary Capacities. My Method is fitted to my deſign: Beginning with the Principles of the Art, and ſo proceeding *gradatim* till I have ſhewed how all ordinary Figures may be Meaſured, Protracted, and Caſt up, without any other Inſtrument of charge but Chains, Compaſſes and Scales. Afterwards for ſuch as deſire higher Attainments, I have endeavoured to ſpeak ſo fully in a little compaſs, of the *Plain Table*, and given ſuch hints, applicable not only to it, but alſo to the *Peractor*, *Theodolite*, and *Semicircle*, as that an Ingenious Perſon may make great uſe of them. But as touching the Doctrine of Triangles, and Tranſmutation of Figures into others equipollent

humbly

with the large Tables (referring to the former) of Logarithms of Numbers, Sines, and Tangents, I I thought it improper cumber this ſmall Manuel, or the unlearned Reader with them, having (as I humbly hope) ſufficiently inform'd him how to find all his Sides and Angles by Inſtruments, and alſo the Content of any Figure without ſuch tranſmutation, reſerving ſuch Curioſities and many others touching Drawing and Painting of Maps, Meaſuring of Ways, and Rivers, *&c.* to a Second Part, which I may perhaps hereafter publiſh upon due encouragement; but if I do not, the Curious may find themſelves good ſtore of work in the Authors even now quoted.

2. It may ſeem ſtrange to ſome, that in referring to the Figures, I ſometimes uſe Words ſeeming to imply that the Figure I ſpeak of is in that very Page, and ſo it was in my Copy, but the Printer and Gravers have otherwiſe contrived them for convenience in Copper Cuts by themſelves. And to give them their due, they are generally done with great accuracy, and none of them having any ſuch error as is like to beget trouble or miſtake to the Reader, ſaving only that fig. 19 hath D inſtead of O at the Center, and the Line O L in the Margin of *Fig.* 14. ſhould be of the length from L to the uppermoſt o in the Scale, and the Figures on the ſide ſhould be made 1 leſs than they are, *viz.* 2 ſhould be 1, 3 made 2, *&c.*

Mr

Mr *COLLINS*
TO TH
READER.

Courteous Reader,

THE Learned Mr *Adam Martindale* formerly Writ two excellent Almanacks, called Country Almanacks, which were Printed, and esteemed by several Members of the Royal Society very useful, especially for Country Affairs, but meeting with some Discouragements from such as knew not how to judge of the Authors worth, he gave over that undertaking, contrary to the desires of many Ingenious Men.

And having since Writ a little Treatise of Survey, in which he hath had Experience, as well as Theory, and being willing to have the Approbation or Dislike of others, that it might either be Printed

Printed or Stifled, imparted the ſame to ſome of the Members of the aforeſaid Illuſtrious Body.

Upon the Peruſal of whom, and divers Experienced Artiſts, who make a Livelihood of it, I find it well approved (as clear and conciſe) only the latter were ſorry ſo much was diſcovered, as detrimental to their Practice, particularly about ſetting off the outjettings, where it's inconvenient intirely to meaſure them.

The Book is ſmall, the price (though not the worth) mean, no ſmall Encouragements to young Students and the Vulgar, for whom it was chiefly intended: By which, that they may reap Benefit, is the hearty deſire of (a Well-willer to the Author and them.)

John Collins.

The

Books Printed for R. Clavel, *at the* Peacock *in St* Pauls *Church-yard, and* T. Sawbridge, *at the* Three Flower-de-luces *in* Little Brittain.

THe Art of Dialling performed *Geometrically* by Scale and Compasses. *Arithmetically* by the Canons of Signs and Tangents. *Instrumentally* by a Trigonal instrument, accommodated with lines for that purpose, the Geometrical part whereof is performed by projecting the Sphere in Plano, upon the Plan itself; whereby not only the making, but the reason also of Dials is discovered; with a second way of Geometrical Dialling, very easy, plain and universal. The Third Edition. To which is added a Supplement with Copper Cuts, Exemplifying the nature of Dials, very useful for all that would attain this Art. By *William Leybourn* Philomath. Quarto.

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THE *Country Survey-Book:* OR, LAND-METER's *VADE-MECUM.*

CHAP. I.

Of Geometrical Definitions, Divisions, and Remarks.

I. A *Point* is that which hath no parts, either of longitude or latitude, but is indivisible, ordinarily expressed with a small prick, like a period at the end of a sentence.

II. A *Line* hath length, but no breadth nor depth, whose limits or extremities are Points. This is either right or crooked.

III. A *right Line* lies ſtraight, and equal between its extreme points, being the ſhorteſt extenſion between them; the *crooked* or *circular* not ſo.

IV. A *Superficies* hath length and breadth, but no depth; of this Lines are the limits.

V. A *plain Superficies* is that which lieth equally (or evenly) between its Lines.

VI. An *Angle* is the meeting of two Lines in one point, ſo as not to make one ſtraight Line, and if drawn on paſt that point, they will interſect or croſs one another. This in vulgar *Engliſh* may be called a *Corner*; of which there be two ſorts, one *right*, the other *oblique*.

VII. A *right Angle* is that which is made by two right lines, croſſing or touching one another perpendicularly, (or ſquarely) like an ordinary Croſs, or Carpenters Square.

VIII. An *oblique Angle* is that which is either greater or leſs than a right Angle, and this is of two ſorts, *obtuſe* and *acute*.

IX. An *obtuſe Angle* is greater than a right Angle, like the left and right Corners of a Roman X.

X. An *acute Angle* is leſs than a right Angle, like the higheſt and loweſt Corners of the ſame Letter.

XI. A *Figure* is that which is comprehended under one line or many: Of this there are two kinds, a *Circle* and a *right-lined Figure*.

XII. A *Circle* is a perfect round Figure, ſuch as is drawn with a pair of Compaſſes, the one Foot being turned round in a point, and the other wheeled about it. The point in the preciſe

cise middle is called the *Center*; the round line, the *Circumference* or *Peripheri*; a line going through the Center, and divide the Circle into two equal parts, is called the *Diameter*; half of that line as a *Semidiameter*, or *Radius*; half the Circle is stiled a *Semicircle*; the quarter, a *Quadrant*; any portion of it, cut off by a right Line not touching the Center, is called a *Segment*.

XIII. *Right-lined Figures* are such as are limited by three right Lines or more, and are either *Triangles* or *Triangulate*, that is, such as are compounded of, and resolvable into Triangles.

XIV. *Triangles* are Figures comprehended under three right Lines, and (as *Ramus* thinks for a Reason that he gives, *lib. 6. pr. 6.*) might be better called *Trilaterals*; but the name *Triangle* from the number of the Angles hath obtained.

Also from the nature and Quantity of their Angles these Triangles are distinguished into three sorts: 1. *Rectangled*, having one right Angle; 2. *Obtuse-angled*, having one obtuse Angle; and 3. *Accute-angled*; having all acute Angles; for no Triangle can have more right or obtuse Angles than one, because by an old Rule (easie to be demonstrated) no Triangle upon a plain Superficies can consist of three greater Angles than such, as being jointly taken are equal to two Right.

These three sorts of Triangles may, according to the length and proportion of their sides, be subdistinguished into seven; for each of them may have either two equal Sides or none, and the Acute-angled may have all three Sides or lines equal: To all which kinds, learned Men

give distinct *Greek* Names, which if mine *English* Reader have a mind to see, they are to be found in Mr *Wing's Geodætos Practicus, Book* 1. *page* 6. for my present purpose the abovementioned trimembred distinction will abundantly suffice; for be Triangles of what name or kind soever, they are all capable of being exactly measured by one plain Rule, as hereafter shall fully appear.

XV. *Triangulate Figures* are such as have more Angles (and consequently more Sides or Lines) than three; and these are either *Quadrangular* or *Multangular*.

XVI. *Quadrangular Figures* are such as have four Angles (and as many Sides) and these are either *Parallellograms* or *Trapezia's*.

XVII. *Parallellograms* are figures that are bounded with parallel Lines, that is, such lines as are every where of the same distance one from another, so as if they were infinitely extended they would never meet, like the upright lines of the Roman H. These *Parallellograms* are either *Rectangular* or *Obliquangular*.

XVIII. *Rectangular Parallellograms* are such as have four right Angles. *viz.* the *Square* or *Quadrat*, and the *long Square*, otherwise called the *Oblong*.

XIX. The *Square* is that Figure that hath four right Angles, and four equal Sides, like any of the six Faces of a Die.

XX. The *long Square* hath also four right Angles, and the opposite Sides are equal, but the adjoyning Sides meeting at each Angle differ in length. Of this Figure is a well Printed Page in a Book, and the Superficies of a well cut

cut Sheet of Paper, or an ordinary Pane of Glaſs.

XXI. *Oblique angled Parallellograms* are ſuch as have oblique Angles, *viz.* two acute, and two obtuſe. Of theſe there are two kinds, the *Rhombus*, and the *Rhombodies*.

XXII. The *Rhombus* is a Figure that hath equal Sides, but no right Angles, (like the form of a Diamond on the Cards, or the moſt ordinary Cut of Glaſs in Windows) whoſe oppoſite Angles are equal.

XXIII. The *Rhomboides* is (as it were) a defective *Rhombus*, for if from any ſide of a *Rhombus* we cut off a part with a parallel Line, the Remainder will be a *Rhomboides*. which hath neither equal Sides nor angles, but yet the oppoſite Sides and Angles are equal.

XXIV. The *Trapezium* is a Figure that is neither parallellogram, nor (conſequently) hath equal Sides or Angles, but is irregulary quadrangular, as if drawn at adventure. Of this ſhape moſt Fields prove, than ſeem to the Eye to be Squares or Oblongs.

XXV. *Multangular Figures* are ſuch as contain more Sides and Angles that four, and they are either *regular* or *irregular*.

XXVI. *Regular Multangulars* take their names from their Number of Angles, ſo a *Pentagon*, *Hexagon*, *Heptagon*, *Octogon*, *Encagon*, *Decagon*, ſignifie multangular Figure of *five*, *ſix*, *ſeven*, *eight*, *nine*, *ten*, Angles and conſequently Sides.

XXVII. An *irregular Polygon* or *multangular Figure*, is that which hath more Angles (and Sides) than four, the Sides (and Angles) being unequal to one another.

CHAP. II.

Of Geometrical Problems.

I. *To draw a Line parallel to another, at any Distance assigned.*

Fig. 1. OPen your Compasses to the Distance given, and chusing two Points conveniently distant in the Line given, as here at A and B describe the Arches G and D, to whose convixity if you apply a Rule, the parallel Line is easily drawn.

II. *To raise a Perpendicular upon a Line given, or to cross that Line at right Angles in a Point assigned.*

Fig. 2. Suppose the point C in the Line A B were assigned for the perpendicular; open the Compasses to a convenient distance, and mark out the two points E and F in the line A B, then opening them somewhat wider, you may (by setting one Foot in E and F severally) describe the two Arches cutting one another at the point D, from which if you draw a Line to the point C, the work is done for the raising of a Perpendicular; but if you be to cross the lines at right Angles, you may continue the line from D thro C at pleasure.

But if the said line A B had been given to be divided in the precise middle, by another Line crossing it at right Angles, the way were to set one point

point of the Compaſſes in A and B ſeverally, and having deſcribed two Arches above the line, interſecting one another as at D, do the like below the line A B from the ſame points, and with the ſame extent of your Compaſſes, then through the ſeveral interſections (a Rule being laid upon them) a line may be drawn, cutting the given line exactly in the middle at right Angles. *Fig.* 2.

Note, That when one point of your Compaſſes ſtand in A, you may make both the Arches belonging to that center above and below the line, and then removing the Compaſſes to B, you may croſs them both.

III. *To raiſe a Perpendicnlur at the End of a Line.*

Let O R be the line given, then to raiſe a Perpendicular at R, make five little equal diviſions, and taking four of them with your Compaſſes ſet one foot of your Compaſſes in R, and with the other deſcribe the Arch P P; then take the diſtance from R to 5, and placing one Foot in 3, with the other deſcribe the Arch B B, interſecting the former in the point S; then ſhall the line S R (being drawn by a ſtraight Rule) be a Perpendicular to the line O R. *Fig.* 3.

IV. *To let fall a Perpendicular upon a given Line from any Point aſſigned.*

Open your Compaſſes ſo as one Foot being ſet in the aſſigned point the other may go clear over the line given, and thereby deſcribe an Arch cutting the Line at two points; then ſhall the half

half distance between those two points be the point to which the Perpendicular may be drawn from the Point assigned. But if you think it too much pains to find the Point of half distance by trial, you may help your self by the second Problem: For if you describe two Arches intersecting one another on the farther side of the line from the assigned point, placing (to that purpose) the Foot of your Compasses first in one of the Intersections of the given line, and then in the other; you may by laying a Rule upon the assigned point, and the intersection of the two Arches, draw a Perpendicular from the said assigned Point, cutting the given Line at right Angles.

☞ *Note*, that all these Problems touching perpendiculars, aim at no greater matter, than what may be performed in a Mechanical way with exactness enough (and much more neatly by avoiding unhandsome Pricks and Arches) by the help of a small Square exactly made (or for want thereof a Plate Quadrant, or broad Rule, having a right Angle and true Sides) for if you apply one Leg of such a Square to any Line, so as the Angle of the Square may touch the end of the said Line, or any other Point where the Perpendicular is to be raised, you may by the other Leg draw the Perpendicular. In like sort to let fall a perpendicular from a point assigned you need only to apply one Leg of the Square to the Line, so as the other may touch (at the same time) the assigned point whence you may draw the perpendicular, by that Leg that toucheth the Point.

If

If the Angle of your Square be a little blunt, either through ill making or long uſing, you muſt allow for it when you apply it to the point in a Line. And when you are drawing a Perpendicular, you muſt ſtop before you reach the given line, and then by applying the Leg of your Square to that part of the perpendicular already drawn, ſo as part of that Leg may paſs clearly over the given Line, you may draw the reſt of your perpendicular as exactly as if the Angle had been true. The like courſe is to be taken when a line is to be croſſed by another drawn quite through it at right Angles.

V. *An Angle being given, to make another equal to it.*

The Angle X A D being given, and a line drawn at pleaſure as is the loweſt from the point E, open your Compaſſes to any convenient diſtance, and ſetting one foot in A deſcribe the Arch B C. Then with the ſame extent ſetting one foot in E, with the other deſcribe the Arch G H, long enough to equal or exceed the other. Then taking the diſtance B C between the points of your Compaſſes, ſet one in G and with the other mark the point H in the Arch G H, through which point H a line being drawn from the point E, will make an Angle with the Line E G equal to the Angle given. *Fig.* 4.

Note, when we ſpeak of the quantity of Angles, their equality or unequality, we never regard the length of the Lines; for if you extend or contract them at pleaſure, the Angle is ſtill

ſtill the ſame. But that is the greateſt Angle whoſe lines are fartheſt diſtant from one another at the ſame diſtance from the Angular Point, or the place where its lines meet.

VI. *Any three Lines being given (equal or unequal) ſo as no one of them be longer than the other two joined together to make a Triangle of them.*

Fig. 5. The Lines A, B, C, being given, ſet the line A from D to E; then with your Compaſſes take the length of the Line B, and ſetting one Foot in D deſcribe the Arch P O. This being done, take with your Compaſſes the length of the line C, and ſetting one foot in E with the other Croſs the former Arch at F, from which Interſection drawing Lines by a Rule to D and E, the Triangle is finiſhed.

☞ *Note*, that if all the Sides, or two of them, be equal, the method is the ſame; but the labour leſs, becauſe we need not to take the ſame length twice over with the Compaſſes.

VII. *To find the Perpendicular of the Triangle, in order to the meaſuring of it.*

Fig. 6. Let the Line A B be accounted the Baſe, and from the Angle C let fall a perpendicular as was taught, *Probl.* 4. upon that line at D, which is ready for taking off with Compaſſes and meaſuring on a Scale, of which hereafter in the Chapters of meaſuring the Content of Figures.

But if we have no occaſion to draw the perpendicular, but only to know the length of it, (as

(as it moſt frequently falls out in meaſuring) no more is needful but to ſet one foot of the Compaſſes in the Angular point C, and extend the other to the Baſe A B, ſo as it may touch it, but not go beyond it; then have we the perpendicular between the points of the Compaſſes.

VIII. *One Side being given, how to make a Square.*

The Line C D being given, raiſe a perpendicular at C of the length (at the leaſt) of the given line; then taking the line C D between the feet of your Compaſſes, ſet it upon the perpendicular from the Angular Point C to A: With the ſame diſtance ſetting one foot in D deſcribe the Arch O P. Laſtly, with the ſame diſtance (or extent) ſet one foot in A, and with the other deſcribe the Arch croſſing the Arch O P in N, from which interſection a line drawn by a rule to A, and another to D, finiſh the Geometrical Square or Quadrant A C D N. *Fig.* 7.

IX. *To make a long Square, the length and breadth being given.*

This is ſo like the former, that a particular Figure is not neceſſary to conceive of it. Suppoſe each ſide of the Square in the laſt Problem to conſiſt of 8 ſmall equal parts, and you were to make a long Square whoſe length muſt be equal to a ſide thereof, *viz.* 8, and the breadth half ſo much given in a line thus ——— 4; then when you had drawn the line C D for the length, and raiſed the Perpendicular at C, you

Fig. 7. you must take the shorter Line given for the breadth, and set it upon the Perpendicular from C upwards to a Point, which for distinction we shall call the Point E, imagining it so marked: With the same extent of the Compasses describe the Arch, placing (to that purpose) one foot in D. Lastly, extending your Compasses to the length of the line C D, set one foot in E, and with the other cross the Arch aforesaid. Then a right Line drawn from that intersection to E, and another from the same to D, compleat the long Square.

X. *To make a Rhombus, the Sides being given.*

Fig. 8. If the Angles be not limited draw any oblique Angle at pleasure, either Accute or Obtuse, as here the Angle B A C, which is Acute. Then let the Line O P be the length of a Side, which being taken with your Compasses, set it from the angular point A in both lines to D and E, in which two points place a foot of your Compasses successively without altering them, *viz.* in D to describe the Arch F G, and in E to describe the Arch H I, crossing one another in the Point K, from which, right lines drawn to D and E, finish the Rhombus D A E K.

☞ *Note*, if any Angle be given, together with the Side; to limit the shape and content, begin with that, and proceed as before: For you must know, that to make a Rhombus (or Rhomboides) like to another for Figure, or equal to it in Content, it is not sufficient to have the same Sides; for the more oblique the Angles

are,

are, the farther will the Rhombus differ from a Square (and the Rhomboides from a long Square) and the leſs will be the Content. But you muſt have an Angle given, (which will produce all the reſt) or elſe a Diagonal Line, which is a right Line paſſing through the Rhombus (or Rhomboides) from one oppoſite Angle to another, and dividing the Figure into two equal Triangles. If the former (*viz.* an Angle) be given, I have ſhewed what uſe is to be made of it. If the latter, (*i. e.* a Diagonal) together with the length of the Sides, you may, by taking the length of the Sides with your Compaſſes, and ſetting a Foot in the ends of the Diagonal Line, make a Triangle on the one ſide of the Diagonal, by *Probl.* 6. and then another on the other ſide by the ſame problem, the Diagonal being a common Baſe to them both; and this will give the Figure exactly.

XI. *To make a Rhomboides, the Sides being given.*

If neither Angle nor Diagonal be given, (for if either of them be limitted, the caſe is ſpoken to in the laſt problem) make any Angle at adventures as here ABC. Then ſuppoſing the Lines given to be OP and QR, ſet the length of the longer upon the Line BC from B to D, and the ſhorter on the Line BA to E. Then with the Compaſſes extended from B to E ſet one Foot in D and deſcribe the Arch F G. Likewiſe, with the Compaſſes extended to the length of the Line O P, ſetting one Foot in E with the other deſcribe the Arch H I, interſecting *Fi.* 9.

Fig. 9 the former Arch at K, from which Interſection Lines drawn to D and E finiſh the Rhomboides.

XII. *To make a Trapezium, the Diagonal and Lines in order being given.*

Fi. 10 Let the Line H L be the Diagonal of a Trapezium, whoſe Sides are the Lines A,B,C,D; the Side A being counted the firſt, as that which takes it being from the point H, and the reſt in the order as they are marked Alphabetically.

Then with your Compaſſes ſet to the length of the Line A, place one Foot in H, and with the other deſcribe the Arch EF. Next taking the length of the Line B, with the one Foot of your Compaſſes placed in L, with the other make the Arch G I, interſecting the former at K, from which Point of interſection, Lines drawn to H and L make the Triangle H K L.

Then with the extent of the Line C, ſet one of the Feet of your Compaſſes at L, and deſcribe the Arch O P. Laſtly, ſetting them to the length of the Line D, and placing one Foot of your Compaſſes in H, with the other make the Arch S R, interſecting the former at Q; ſo ſhall Lines drawn from Q, to L and H, make up the Triangle L Q H, and finiſh the Trapezium H K L Q.

I could have been much briefer in this problem by referring to the ſixth; but this being of very great and frequent uſe, I deſired to be very plain.

XIII. *To*

XIII. *To make a regular Polygon, otherwiſe called a regular multangular, or multilateral Figure, conſiſting of many equal Sides and Angles,* viz. *above four apiece.*

Being ſatisfied what ſhall be the diſtance between the Center and every Angle, with that diſtance deſcribe a Circle, which being equally divided into as many Parts, as the Figure muſt have Angles (or Sides, for they are equal in number) and Lines drawn from the Points of Diviſion within the Circle from Point to point, (ordinarily called Chords) the Polygon is finiſhed as in this Diagram.

Suppoſe an Heptagon, or multangular Figure of ſeven Sides, and as many Angles, be to be deſcribed, every Angle being deſigned to be diſtant from the Center A, ſeven Eighths, or three quarters and a half of an Inch; with that diſtance deſcribe the Circle B C D E F G H, which being divided into ſeven equal parts, and Lines drawn from Point to Point, the Heptagon B C D E F G H will be therein included. *Fig.* 11.

I ſhall rather leave my unlearned Reader to find out the Points of Diviſion by many tryals, than to puzzle him with the Geometrical way for finding out Chords to that purpoſe; nor ſhall I buſie my ſelf to tell him at large how he may divide 360 by the number of his Angles or Sides, and then finding in his Quotient the Degrees and parts belonging to every Diviſion, ſet them readily out by a Protracter, or (for want thereof)

by a Line of Chords; for I ſuppoſe him yet ignorant of ſuch things.

I ſhall therefore only tell him thus much: A Line drawn through the Circle at the Center divides it into two equal Parts, which being croſſed in the Center by another Line, the Circle will be parted into four equal parts or Quadrants, and thoſe by halving them into eight Parts. The extent of the Compaſſes whereby the Circle is drawn (uſually called the Radius or Semidiameter) will divide it into ſix equal parts; two whereof muſt be a third part, and half of one a twelfth part; and theſe ſtill eaſily capable of farther Diviſion.

XIV. *Having the ſides of the Triangles whereof it conſiſteth, orderly given, to make an irregular multangular, or multilateral Figure.*

This will be more fully handled hereafter, when I come to ſhew the method of drawing plats of Ground: In the interim I will give you a Specimen of an irregular Pentagon.

Having the Lines of three Triangles given, *Fig.* 12 (which by a Rule hereafter to be mentioned are neceſſary to make up a five ſided Figure) lay down the greateſt of the firſt, *viz.* 20, from A to B for a Baſe, and by *Probl.* 6. make a Triangle of it and the other Lines 16 and 10, *viz.* the Triangle A B O.

Secondly, you find by the number 20 over the firſt Line of the ſecond Triangle that it is the common Baſe to them both, and therefore by the ſame *Probl.* 6. make the Triangle A B P of the Lines 20, 14, 18. Laſtly,

Laſtly, finding the Baſe of the third Tri- *Fi.* 12.
angle to be the ſame with 18, one of the Sides of the ſecond, make the Triangle P B Q of the Lines 18, 11, 12: So is the quinquangular Figure finiſhed.

How every Line is to be found in it's due order in this or any other ſort of multangular Figures ſo as to give a true and exact account, not only of the ſuperficial Content, but alſo of the Figure (or ſhape) and ſituation, is to be taught hereafter in the Doctrine and Practice of protraction.

CHAP. III.

How to find the Superficial Content of any Right-lined Figure, the Lines being given.

AS a Foundation to what I ſhall ſay upon this Subject, there are ſome few Geometrical Principles or Theorimes out of *Euclid* and *Ramus*, which I deſire may be remembred; and becauſe underſtanding is a mighty help to memory, I deſign for my Country Reader a kind of ocular Demonſtration, which though not ſo ſtrict and artificial as that which is to be found in the Commentators upon *Euclid* in the quoted places, will be more ſerviceable to him, becauſe more eaſily underſtood.

Theor. 1. *Every Parallelogram being of the same length with the Base of a Triangle, and of the same heighth with the Perpendicular of that Triangle, is double to it,* Euclid 41. 1.

Fig. 13 Here are two equal Oblongs or long Squares, A B C D and B E F C, and within them two Triangles inscribed, whose Bases are of the same length, and their prependiculars (O P and Q R) of the same heighth with the Oblongs. Now each of these Triangles, being parted into two Right-angled Triangles by their Perpendiculars, then it is plain to the Eye (and from the nature of Diagonals, which ever divide a Parallellogram into equal parts) that the two new Triangles O P D and O P C, which make up the first of the given Triangles, are equal to the Trangles D A O and O B C, which make up the remainder of the Parallellogram A B C D. Therefore that Parallellogram is double to that Triangle, which was to be demonstrated.

In like manner it is evident, that the Parallellogram B E F C is double to the Triangle C O F, becaufe C R Q is equal to B Q C, and Q R F is equal to Q E F.

Theor. 2. *All Triangles having the same Base, and lying between the same Parallels, are equal.* Euclid 37. 1.

So in our last Diagram, the two given Triangles having Bases of the same length, and lying

lying between the ſame parallels, are evidently equal, becauſe they are demonſtrated to contain each of them the exact half of the Parallellograms, wherein they are inſcribed; and the Parallellograms being equal, their halves muſt be equal alſo.

Theor. 3. *The ſides of the Triangulate (that is, one that hath four or more Sides) are ever two more than the Triangles of which it is made.* Ram. lib. 10. prop. 2.

This is plain by inſpection, if you view again the Figures of the Trapezium and irregular Pentagon, in the 12th and 14th Problems of the ſecond Chapter. *Fi.* 10, 12.

Theſe Theorems being allowed to be ſound (as nothing more certain) the Doctrine concerning the Superfi ial Content of Right-lined Figures might be reduced to a narrow compaſs; for he that knoweth how to husband theſe three Theorems, may eaſily take up theſe *Corollaries, ordine inverſo.*

1. Any quadrangular Figure (regular or irregular) may by a Diagonal be parted into two Triangles; any five-ſided Figure by two Diagonals into three Triangles; and ſix-ſided Figures into four by three Diagonals, *&c.* by *Theor.* 3.

2. 'Tis no matter of what ſhape the Triangle is, as to the rule for meaſuring, for whether it be Right-angled, Acute-angled, or Obtuſe-angled, and whether it have three, two, or no Lines equal; 'tis only the length of the Baſe, and

and height of the Perpendicular, and is consi derable by *Theor.* 2.

3. The true measure or content of any Tri-angle whether alone, or as part of any triangulate Figure of 4, 5, 6, or more Sides (and consequently of the whole Figure, by summing up the content of all the several Triangles) is found by *multiplying the whole Base* of the Triangle by *half the Perpendicular*, or the *whole Perpendicular* by *half the Base*; which being a Rule of such infinite use in surveying, I desire it may be remembered; and that it may be Understood I shall give you a plain Example.

Fig. 6. Suppose A B the Base of the Triangle, belonging to the seventh Problem of the foregoing Chapter, to be 44, and the Perpendicular C D to be 20: Whether you multiply 44 the whole Base by 10 the half of the Perpendicular, or 20 the whole Perendicular by 22 the half Base, the product gives the Content 440, as is here apparent.

44	20
10	22
440	40
	40
	440

Having thus given a general method how all Right-lined Figures may be reduced to Triangles, and so their Content found out; I might pass to the next Head concerning Instruments, and their use; but because there are nearer ways in measuring particular kinds of triangulate Figures proper to those kinds, I shall briefly touch them.

I. *To find the Contents of a Square, or long Square.*

Multiply the length by the breadth, the product gives the *Area* or Content.

Example of a Square.

17 Inches length.
17 Inches breadth.
119
17
289 Square Inches

Example of an Oblong.

25 Feet long.
13 Feet broad.
75
25
325 Square Feet.

II. *To find the Area, or Content in Measure of a Rhombus.*

Let fall a perpendicular from one of the obtuse Angles upon the opposite side; that side multiplied by the perpendicular, gives the *Area*.

20 Yards the Side.
14 Yards the Perpendicular.
80
20
280 Square Yards.

III. *To*

III. *To find the* Area *of a Rhomboides.*

Divide it into two Triangles by a Diagonal drawn between either pair of the oppoſite Angles, (as ſuppoſe the Acute) then from either of the other Angles (for inſtance, the Obtuſe) let fall a perpendicular upon that Diagonal; then ſhall that Diagonal, being multiplied by that perpendicular, give the *Area*.

Example.

19 Rods the Diagonal.
5 Rods the Perpendicular.
95 Square Rods the Content.

IV. *To find the Content of a Trapezium.*

Fi. 10. Divide it by a Diagonal into two parts from Angle to Angle; as the Trapezium, *Ch. 2 pr.* 12. is divided by the Diagonal H L, then from the other two Angles (which in that Figure are marked with K and Q) let fall Perpendiculars upon the Diagonal, half the Sum of thoſe Perpendiculars being multiplyed by the Diagonal (or common Baſe) gives the Superficial Content.

Example.

Suppoſe in the Trapezium before mentioned the Diagonal is 33, the Perpendicular from K 13,

13, and that from Q 15, the *Area* or Superfi- *Fi.* 10.
cial Content is thus Computed.

13 Chains the first Perpendicular.
15 The second Perpendicular.

28 The Sum of both Perpendiculars.

14 Their half Sum.
33 The Diagonal.

4^2
462 Square Chains the *Area*.

V. *To find the Content of a regular Polygonial, or multangular Figure, otherwise called multilateral.*

Draw a Line from the Center to the middle of any side; half of the Perimeter (or of all the Sides) being multiplied by that Line beforementioned, gives the Content.

VI. *To find the Content of an irregular Polygon, or many sided Figure.*

Divide it into Trapezia's and Triangles by Diagonal, then find their Content severally, and sum up all together; which that you may better apprehend,

Suppose the Polygon belonging to the last *Fi.* 12.
Problem of the second Chapter were given without the Diagonals A B and B P; then by drawing

§. 12. ing those Diagonals, the Figure is divided into three Triangles, whereof two being upon the same Base A B make up a Trapezium, whose Content may be found, just in the same manner as was taught even now in the fourth Rule, having found the perpendiculars from O and P falling upon the Line A B. Then there remains the Triangle B Q P, whose Content may be found by the general Rule concerning Triangles, having found the Perpendicular falling from Q on the Line BP; and then having added the Content of that Triangle to the Content of Trapezium, you have the *Area* of the whole polygonal Figure.

13 But now methinks I see (as it were) my Country Student scratching his head, and wishing for an opportunity to propound two doubts to me.

1. Why I called the Numbers correspondent to my Lines by divers denominations, as Inches, Feet, Yards, Rods; and sometimes by none at all, but propounding the Numbers abstractly.

2. How I came to know how many of those Measures (whatever they be) are represented by the Lines given, and perpendiculars found.

To the former I answer, I am not yet teaching how to measure Lines (that work is presently to follow) but what Lines of Figures are to be measured, and the measures of those Lines being known (or supposed) how the Content upon those (real (or supposed) Grounds may be found, and to this purpose I might call the Numbers represented by the Lines, Inches, Feet, Yards, or any other Measures, at pleasure, pro-

provided I called the Squares, to which the *Area* is equal, by the ſame names; for an Inch in length bears the ſame proportion to a ſquare Inch (having length and breadth) that a Mile in length bears to a ſquare Mile. For this reaſon I profeſs not in the Title of this Chapter to teach how to meaſure Figures, much leſs how to meaſure the Lines of ſuch Figures, but how to find the Content, the Lines being given.

And then to take away the ſecond doubt, know that the numbers repreſented by the Lines, were either given by thoſe Learned Artiſts from whom I borrowed the Figures, or ſuppoſed by my ſelf as grounds to go upon (as in ſuch caſes is ordinary) or laſtly, found out to be agreeable thereto by ſome Scale of ſmall equal parts, which he is yet ſuppoſed ignorant of. But now I am going to ſhew him the nature and uſe of two or three plain and cheap inſtruments, by the help whereof he may with much exactneſs 1. Meaſure the length of any Lines bounding Right-lined Figures upon the ground. 2. Draw Lines and Figures upon paper proportionable thereunto (which we call protracting.) 3. Find upon his paper-figures the true length of all the deſired Perpendiculars, which ſhall alſo be proportionable to thoſe on the Ground, but much more eaſie to be obtained. And withal, I intend to give him ſuch farther Inſtructions and Cautions for the application for the general and more particular Rules of this Chapter to his peculiar uſe, as will render them (eſpecially ſome of them) ſingularly advantageous.

CHAP. IV.

Concerning Chains, Compasses and Seales.

1. AMongst the many sorts of *Chains* used for measuring Land, three are most famous, bearing the Names of their Inventors, Mr. *Rathborne*, Mr. *Gunter*, and Mr. *Wing*, all of them ingeniously divided, and useful in their kind; but my brevity will give me leave only to describe one, and that shall be Mr *Gunter's*, being most in use, and easy to be procured.

This Chain contains in length four Statute-Poles or Perches, each Perch containing 16 Feet and a half, or 5 Yards and a half; so that the whole Chain is 66 Feet, or 22 Yards long.

This whole Chain is divided into 100 equal parts or Links, whereof 25 are a just Pole or Perch; and for ready counting there is usually a remarkable distinction by some Plate or large Ring at the end of every 25 Links, but especially at the precise middle of the Chain, which should differ from the rest in greatness and conspicuousness. Also at the end of every tenth Link 'tis usual to hang a small Curtain Ring; and if there be at every five Links end a piece of Wire made like the bow of a Link, with a little shank an Inch or less long, (or some such distinction) 'tis still better.

When you are to measure any Line by this Chain, you need to regard no other Denomination

nation but only Chains and Links, ſet down with a prick of your Pen betwixt them, *e. g.* If you found the ſide of a Cloſe to be 6 Chains and 35 Links long, it is thus to be put down 6 . 35.

But if the Links be under 10, a Cypher muſt be prefixed; ſo 7 Chains 9 Links muſt be thus ſet, 7 . 09.

In the uſing of this (or indeed of any) Chain care muſt be taken, both to go ſtrait, and to keep a true account; for which purpoſe, it is good that he which goeth before carry in his hand a bundle of Rods, to ſtick down one at the end of the Chain which leads, having firſt ſtretcht it well, and that he which follows do not only gather up the Rods to keep the Accompt, but alſo at every remove, mark whether he ſee the Leader directly between his Eye and the Angle, or other Mark he aims to meaſure to; and if need be, call to the Leader to move towards the right or left hand, till he ſee him in a direct Line to it.

II. *Compaſſes* are ſo well known, that I need not deſcribe them; only they ſhould be of Braſs, with Steel points ſmall and neatly wrought, nine or ten inches long from the Joint to the points, turning ſo truly upon the Rivet that they may be eaſily opened; and yet ſtand ſo firmly, that an Arch or Circle may be without their ſhrinking deſcribed upon a large *Radius*.

For the form, I would commend above all, thoſe that have large Bows, ſo contrived that by preſſing them with the hinder part of the Hand, they will gently open, and by the Thumb

and fourth Finger be put together (as others will) ſo that they are manageable by one hand, which is a great convenience for one that at the ſame time ſhould hold his Rule with Scales in the other.

Theſe might alſo be contrived with a Screw to take out one of the points, to place in the room (upon occaſion) a Black-Lead pen, or any pen to draw Circles with, either black or otherwiſe coloured.

And for a Man that would be an Artiſt indeed, it were convenient he were furniſhed with dividing Compaſſes, beam Compaſſes, and triangular ones, for ſevel uſes not here to be mentioned; but a Country Surveyor may make a good ſhift with ſuch a plain pair as I firſt deſcribed, which Mr *Wynn*, over againſt the *Rolls* in *Chancery-lane*, will help him to, with the Chain and Scales for a ſmall matter.

III. *Scales* are certain Lines divided into equal parts, upon plates or broad Rules of Braſs or Box, and they are of two ſorts, 1. Plain; 2. Diagonal.

1. *Plain Scales* are made up of two ſmall Lines parallel to one another at a little diſtance, and theſe are divided into great equal parts, which ſignifie Tens, and are noted 10, 20, 30, 40, 50, *&c.* according to the length of the Lines.

They may be of any convenient length, but theſe great diviſions are ſeldom more than Inches, or leſs than third parts apiece.

Again, one of the great Diviſions (or parts) is ſubdivided into ten equal parts by ſhort Lines, whereof

whereof that in the middle ſtanding for 5 is longer than the reſt.

According to the numbers of theſe little parts contained in an Inch, the Scale is named *A Scale of* 10, 11, 12, 16, 20, 24, 30, &c. *in an Inch.* That ſhort one which I give you the Figure of A is of 10 in an Inch, ſo noted at the top, according as is uſual upon Rules, and Indices of plain Tables. The line marked O Q, ſeparates Unites and Tens; Unites being taken upward from that Line, and Tens downward; mixt Numbers both ways. *Fig.* 13.

As for Example.

7 is the extent of the Compaſſes upon the Scale A from the Line O Q to K; 30 is their extent from the Line ſo marked to O Q; and 27 is their extent from the Line 20, to the ſhort Line K aforeſaid.

Here note, that you muſt not expect to find the Letters O Q or K upon the Scales which you buy, being only marks uſed at pleaſure, to make my meaning plain; and likewiſe that this Scale of 10 in an Inch, and others that are ſmaller (all being compoſed after the ſame manner) are uſually made for more convenient uſe, ſo long as to contain nine or ten (the more the better) of the great Diviſions, ſignifying Tens; though the Figure at A being deſigned for no other uſe than to help our conceptions, extends but a little beyond 30, that length being ſufficient for my purpoſe in this place.

Fig. 14. Theſe plain Scales, eſpecially the ſmaller ſorts of them (ſuch as 24 or 30 in an Inch) are very proper for drawing Figures upon paper, where the Numbers repreſented by the Lines are not above 100 (for then every Diviſion may be counted as it is upon the Scale) or above upon a long Scale.

Alſo in the ſurveying of Foreſts, Chaſes, and great Commons, where the Lines are vaſtly long, and the miſtake of a few Lines) yea. of half a Pole) is not conſiderable, they may be conveniently uſed, accounting the Tens and Unites to ſignifie ſo many whole Chains, and ſo eſtimating the parts of a Chain with the Compaſſes upon the ſmall Diviſions, which a ſagacious Man may do very near upon one of the larger Scales. But it were much better, in my opinion, for ordinary meaſuring, if the grand Diviſions on the Scale were two Inches apiece, as I have one upon the Index of my plain Table) for then the ſmaller Diviſions being of five in an Inch, would be ſo large as to be ſubdivided into five apeice, which repreſents 20 Links; and then the half of one of thoſe ſmaller Diviſions ſignifying 10 Links, and the quarter 5, a very ordinary judgment may come very near to the truth by eſtimation.

2. But the *Diagonal Scale* is ſo well known to every Mathematical Inſtrument-maker, ſo eaſie to be procured, and every ways ſo fitted to *Gunter*'s Chain, and our Countryman's uſe, that I cannot but highly commend it.

Of theſe Diagonal Scales, there are two ſorts the Old and New.

B

By the Old one, I mean ſuch as is to be found in Mr *Leybourn*'s Book, whereof I ſhall preſent you with a fragment, with ſuch a deſcription as may enable you to underſtand the whole. *Fig.* 14

1. It is made (as appears by the Figure B) upon eleven parallel Lines equidiſtant, ſo as to include ten equal ſpaces, which are all cut at right Angles by Tranſverſe Lines dividing them all into four equal parts.

2. One of theſe Tranſverſe Lines (*viz.* P R) where it toucheth the firſt and laſt Lines, ſeparates between the Hundreds (or whole Chains) and the Tens, repreſenting 10 Links apiece) the Chains being numbered downwards on the left hand from P only to 3, but on the Inſtrument it ſelf they may go on to 9 or 10, (the Rule being a Foot long) but the Tens (or Decads) upward from P to 10.

3. From the Points of Diviſion into Tens upon the firſt Line beginning at P, to the like Points beginning at R in the laſt Line, are nine Diagonal Lines drawn, the firſt beginning at P and ending at the firſt Diviſion above R. The ſecond beginning at the firſt Diviſion above P, and ending at the ſecond above R. In a word, they are all drawn from one Diviſion leſs from P, to one more from R; by which it comes to paſs that every Diagonal, by that time it hath paſſed from the firſt Line to the eleventh is a whole tenth part of an Inch (which anſwers to ten Links of the Chain) farther diſtant from the Line P R, than at the Point upon the firſt Line whence it was drawn.

4. Every

Fig. 14. 4. Every one of those Diagonals is divide into ten equal parts by the long parallel Lines running through the whole Scale, and numbred on the top from 1 to 9. Whereby it is evident, that the Intersection of any of the nine parallel Lines that are numbred at the head with any Diagonal, must be farther distant from the Line P R, than the Intersection of the Line next before it with the same Diagonal by $\frac{1}{10}$ of $\frac{1}{10}$ that is, by $\frac{1}{100}$ which answereth to a single Link of your Chain.

From what hath been said, and inspection of the Figure B, these things plainly follow, which as so many clear instances will help you to understand it fully.

1. The distance from P R to the second Division below it answereth to two Chains.

2. The distance from P R to the eighth Division upward being taken (with Compasses) upon the first Line of the eleven from P to 8, answereth to 80 Links.

3. Consequently the extent of the Compasses from the second grand Division below P to the eighth of the less Divisions upward, is proportionable to 2 Chains 80 Links.

4. The distance from P R to the first Diagonal being taken upon the parallel Line noted with 9 above answereth to 9 Links: Where note, that the first Diagonal is not that which is noted with 1, but that which is drawn from the point P.

5. The distance upon the same Line from PR to the Diagonal that is marked with 7, is answerable to 79 Links.

6. The

6. The extent of the Compaſſes from the bottom of the Figure B upon the ſame Line to the ſame Diagonal, anſwereth to 3 Chains 79 Links. *Fig.* 14.

Briefly whole Chains may (by Analogy) be meaſured upon any Line from P R to the grand Diviſion noted with the given Number, Decads alone, or Chains and Decads upon the firſt Line of the eleven where the Diagonals begin. Links alone, Decads with Links, and Chains and Decads with Links, always upon that Line upon which the number of odd Links ſtands at the head of the Scale.

And know, that theſe Directions (*mutatis mutandis*) will as well fit, if half an Inch be only allowed for a Chain, and conſequently all the Diagonals drawn within that extent, as it is uſual (and very commodious for longer Lines) upon the other end of the ſame rule, the grand Diviſion for Chains going the contrary way, and noted with Numeral figures in order. It is good therefore when you furniſh your ſelf with Scales, to have Diagonal Scales of both dimenſion on the fore-ſide of your Rule; and upon the back-ſide many plain Scales of equal parts, with a Line of Chords; all which you may have (by enquiring only for the Scales deſcribed in Mr. *Leybourn*'s Book) of Mr. *Wynne* aforeſaid, as likewiſe all other Mathematical Inſtruments.

Having been ſo large for my plain Countryman's ſake, I ſhall not proceed to the diſcription of the new Diagonal Scale, of which you may have the Figure and Deſcription in Mr *Wing*'s Book: For though it be an excellent good one,

as

as I know by experience; Mr. *Hayes* having (at my desire) furnished my noble Friend Sir *Charles Hogton* with an artificial one of that sort, when I had the honour of assisting him in Mathematical Studies) yet because 'tis pretty costly (if well made), and that before described will very well answer its end, I shall at present say no more of it.

But my Reader may perhaps object to me, that though I have instructed him how he may make a Line of an exact length, to answer to any number of Chains and Links (given or found by measure) upon the Diagonal Scales: I have not yet shewed him how to measure a Line (as suppose a Perpendicular) whose length is unknown, upon them.

To give him therefore all satisfaction (though what I have writ already might help him to find this out) let us suppose, that in some Figure made according to the Diagonal Scale B of 100 in an Inch, we meet in measuring with an unknown Perpendicular equal to the Line in the Margin. Taking it between the Points of my Compasses, I first try whether it be even Chains, and finding upon the first view that it is not, I make a second trial, whether it will prove to be even Decads, or Ten of Links; to which purpose I set one Foot at 3 Chains in the bottom of my Scale in the first Line where the Diagonals begin, and the other Foot rests in the same line betwixt 6 and 7; whereby I am assured, the odd Links above 3 Chains are more than 60, and less than 70. And to find how many above 60, I remove the Compasses from parallel to

parallel

parallel in order, till one Foot in the loweſt Line reſting in the end of a parallel, the other will touch ſome Diagonal at the Interſection with that line, which falls out to be at L and O in the line marked with 7, ſhewing the whole line, being meaſured by that Scale, to ſignifie 3 Chains and 67 Links. *Fig.* 14.

CHAP. V.

How to caſt up the Content of a Figure, the Lines being given in Chains and Links.

HAving deſcribed theſe plain Inſtruments, and in ſome meaſure ſhewed theuſe of them in ſeverals, it were very proper in the next place to teach their joynt uſe in meaſuring and protracting; but becauſe I would have my young Surveyor, before I take him into a Cloſet, able to perform his whole work together, I intend to ſhew him, 1. How he ought to make his Computations; 2. The Grounds or Principles that will juſtifie him in ſo doing.

For the firſt, take theſe Rules:

1. Put down your length and breadth of Squares and Oblongs, and your Baſe and half Perpendicular of Triangles directly under one another, expreſſed by chains and links with a prick betwixt them, as was taught before, *Chap* 4.

2. If the odd links were under ten, put a Cypher before the numeral Figure expreſſing them

them, (as there alſo was ſhewed) and if their be no odd Links, but all even chains, put two Cyphers after the prick.

3. Multiply length by breadth, and Baſe by the half Perpendicular, according to the Rules for finding the content of Figures, *Chap.* 3.

4. From their Product cut off 5 Figures) accounting Cyphers for ſuch) reckoned from the right hand backward, with a daſh of your Pen, ſo ſhall thoſe to the left hand ſignifie Acres.

5. If thoſe five cut off were not all Cyphers, multiply them by 4, and cutting off five toward the right hand again, the reſt will be Roods or Quarters.

6. If amongſt theſe five Figures towards the right hand that were cut off at the ſecond Multiplication there be any Figures beſides Cyphers, multiply all the five by 40, and cutting off five again by a daſh of your Pen, thoſe on the left hand ſignifie ſquare Perches, Poles, or Roods.

A few Examples will make all plain.

Queſt. 1. *What is the content of a Square, whoſe Sides are every one of them 7 Chains, 25 Links?*

Length 7. 25.
Breadth 7. 25

3625
1450
5075

5125625

5125625

5 25625
4

1 02500
40

1 00000

Answ. 5 Acres, 1 Rood, and 1 Perch, as here appears.

Quest. 2. *In a long Sqare, whose length is* 14 *Chains, and the breadth* 6 *Chains* 5 *Links, what is contained?*

Length 14.00
Breadth 6.05

7000
84000

8 47000
4

1 88000
40

35 20000

Answ. 8 Acres, 1 Rood, and 35 Perches, as the Work makes it evident.

Quest. 3. *In a Triangle, whose Base is* 3 *Chains and half the Perpendicular* 98 *Links, what is the Content?*

The Base	3.00
Half Perpend.	0.98
	2400
	2700
	129400
	4
	1	17600
	40
	7	04000

Answ. 0 Acres, 1 Rood, 7 Perches, as here is plain.

There be no other ways of Computation by Scales, Tables, *&c.* but that this is sound and demonstrative, I come now to shew by these following Steps.

1. It is evident, that in this way of Multiplication the product is square Links; for every Chain being 100 Links, it is all one to multiply 7.25 by 7.25, or 725 by 725 without pricks, for the pricks signifie something as to Conception, but nothing at all in Operation. The Product therefore of the first Example was really 525625 Links.

2. Every Chain being 4 Perches long, it follows, that 5 Chains (or 20 Perches) in length,

length, and 2 Chains (or 8 Perches) in breadth, make an Acre, or 160 ſquare Perches; for 20 being multiplied by 8, gives 160.

3. From hence it plainly followeth farther, that there are exactly 100000 ſquare Links in an Acre; for 5 Chains multiplied by 2, is the ſame with 500 Links by 200, which makes 100000. And he deſerveth not the name of an *Arithmetician* that is ignorant of this old plain Rule, *When the Deviſor conſiſts of* 1 *and Cyphers*, (*as* 10, 100, 1000, 10000, 100000, &c.) *cut off from the right hand ſo many Figures of the Dividend as the Deviſor hath Cyphers, accounting them the Remain; ſo ſhall the reſt on the left ſide be the Quotient.* It is plain then that 525625 ſquare Links make 5 Acres, and 25625 ſquare Links over.

Thus I have made clear to a very ordinary capacity, that as far as concern Acres, the Rules for Computation are good. Now for Roods and Perches, though I might turn off my Reader with that known Rule in Decimal Arithmetick: *Multiplying Decimal Fractions by known Parts, gives thoſe known Parts in Integers, due regard being had to the ſeparation.* I ſhall proceed in my plain way thus. If 25625 ſquare Links, which remain above an Acre, do contain any quarter or quarters of an Acre; then if they be multiplyed by 4, and divided by 100000, (that is, five cut off from the Product) they will contain ſo many Acres as now they do quarters (or Roods,) for any number of quarters multiplyed by 4, muſt needs produce the like number of Unites or Integers, and the Diviſion doth only reduce them into the right denomina-

tion. Now 25625 being multiplyed by 4, and five Figures being cut off from the product, the result is, 1102500, that is an Acre and above; which shews it was above a quarter before it was multiplied by 4.

And to find how much, (that is, now many square Perches are contained in this last remainder (you must consider this 2500, not as square Links remaining above the Rood or Quarter, but as fourth parts or quarters of square Links; or (which is all one) as the true number of square Links multiplied by 4, and consequently being multiplied by 40, the fourth part of square Perches in an Acre) it must as often contain 100000 square Links (or an Acre) as the quarter of this number 2500, *viz.* 625, signifying square Links, containing square Perches; and so it doth, for 100000 divided by 160 (the number of Perches in an Acre) gives 625 as answerable to 1 Perch; and 2500 multiplied by 40, gives 100000, or 1 Acre, the five Cyphers being cut off, as here is manifest.

2500	160	100 ccc 625
40		960
1100000		400
		320
		800
		800
		0

Some

Some may perhaps wonder, that in ſo ſmall a Manual I ſpend ſo many words about ſuch ordinary things, as in this and the laſt Chapter; but I am moſt afraid, leſt I ſhall not for all my plainneſs be ſufficiently underſtood by ſuch as I purpoſely write for, in things of ſuch neceſſary and frequent uſe: And I deſigned not to make this Treatiſe ſmall by being obſcurely brief in ſubſtantial things, but by leaving out ſuch Curioſities as I thought my Country Friend might well ſpare.

CHAP. VI.

How to meaſure a Cloſe, or parcel of Land, and to Protract it, and give up the Content.

HItherto we have been like Children learning to ſpell, now let us ſet our Syllables together. I mean, let us make uſe of the Inſtructions beforegoing to meaſure a piece of land, to plot it, and to caſt up the Content.

All Cloſes, or parcels of land, are either ſuch as need not to be plotted for finding out their true meaſure, but the Chain alone doth the Work; or ſuch as cannot be conveniently meaſured without plotting or protraction.

Of the firſt ſort are the Square and long Square, known before-hand to be ſuch, or found ſo to be by ſuch Inſtruments as I have not yet

described, or by measuing all the Sides and Diagonals. These Squares and long Squares (I say) need no protracting, for you need only to multiply the Chains and Links of the length, by the Chains and Links of the breadth, and so proceed as in the first and second Examples of the fifth Chapter: But all others, whether Triangles or Triangulate, are to be protracted. I shall give Examples therefore in the 3 sorts of Figures, triangular, quadrangular, and multangular.

But before I proceed to particular Instances let me advise the young Practitioner thus: Remember,

1. To begin at some notable Angle of the Field, where there is some House, Gate, Stile, Well, or the like; or if there be none, then to dig up a Clod, drive down a Stake; or at least to observe what quarter of the Heavens it pointeth towards, whether East, West, North or South, and on your Paper mark it with the Letter A, or any other.

2. To go parallel to the side of the Field, if Pits, Bushes, or the like, hinder not, (and if they do, to allow for it) accustoming your self to go either *Cum Sole*, that is, with your left hand towads the Hedges, Walls, or Pales; or *contra Solem*, with your right hand towards them: and when you go contrary to your usual custom, note it on your paper by some mark known to your self.

3. To set down the Chains and Links of every side as you measure them, and not trust your memory. A Black Lead pen will be very proper for this purpose.

4. T

4. To take heed (if you have more Scales than one upon your Rule) lest you confound your self by taking lines off of several Scales, or measuring perpendiculars upon wrong ones; for every line of the same Figure must be made by the same Scale, and the perpendiculars measured by it.

5. To make use of a Scale of larger Divisions when you measure small Closes, and of smaller when you measure great ones.

6. To make your lines and points where Angles meet, small, pure, and neat.

7. To set on your Chains and Links at twice, when any line is too long for your Scale.

These things being premised, I proceed thus:

I. Suppose I measure a triangular Field with my Chain, beginning at the Eastern Angle A, and find the Sides in their order and measure to be severally, thus: (I going *cum Sole*) 2.29 3.45, 4.07.

Making use of the less Diagonal Scale, because the other would make the Figure too large, otherwise it were more proper for so small a Close) I first with my Compasses take off the Scale 4 Chains and 7 Links, and setting them from A to C draw that line for the Base, because the longest of the three: Then I take 2 Chains, 29 Links off the same Scale, and set them in the Eastern point A where I began, and turning the loose Foot of the Compasses above the line A C, because I went *cum Sole*, I describe (at that distance 2. 29.) the Arch E E. *Fig.* 15.

Next

Fig. 15. Next taking with my Compasses upon the same Scale the extent of 3 Chains 45 Links, I place one Foot in the point C, and with the other make the Arch FF intersecting the former in the point B; and drawing the lines A B and B C, (as was taught in *Ch.* 2. *Probl.* 6.) the Triangle A B C is the Plot of the Triangular Field measured.

But before I can give the Content, I must find the length of the Perpendicular, which is done by setting one Foot of the Compasses in B, and extending the other to the Base A C, so as it touch it and pass not over it (according to *Ch.* 2. *Probl.* 7.) for then the length of the Perpendicular is between the Points of the Compasses, and being applied to the same Scale by which the Triangle A B C was made, it appears to be 1 Chain 42 Links. With the half whereof I multiply 4.07 the length of the Base, and proceeding in my Work as was shewed in the last Chapter, the Content appears to be 0 Acres, 1 Rood, 6 Perches, as it is here evident.

The Base	4.07
Half Perpend.	0.71
	407
	2849
	128897
	4
	1)115588
	1)115588

1111 5588.
40

6123520

II. Suppoſe I were to meaſure a quadrangular or four-corner'd Field, I begin as before at ſome remarkable Angle; and going round the Cloſe *cum Sole*, I find the ſides to be 9.04, 6.72, 8.46, 7.28. and the Diagonal from that remarkable Angle to the oppoſite Angle to be 10.02, I begin therefore to protract it thus.

Having by the help of my Scale and Compaſſes drawn my Diagonal 10.02 from my remarkable Angle A to C the oppoſite Angle, I make a Triangle of it, and the firſt and ſecond Sides 9.04 and 6.72. according to *Ch.* 2. *Probl.* 6. and another after the ſame method of that Diagonal, and the third and fourth Sides 8.46 and 7.28, ſo have I the Trapezium ABCD. *Fig.* 16.

Then by the help of my Scale and Compaſſes, I find the Perpendicular of the Triangle A B C to be 6.02, and of the other, *viz.* CDA 6.01, which added are 12.03. whereof the half Sum is 6.01; by which multiplying the Baſe 10.02 and proceeding as formerly hath been ſhewn, I find the Content of the Field to be 6 Acres, 0 Roods, 3 Perches, as is here apparent.

The

Fig. 16

The Base 10.02
Half Perpend. 6.01

1002
60120
6102202
4
108808
40
3152320

Before I pass any further, let me tell you
1. Any quadrangular Close, or parcel o
Ground whatsoever, having right Lines, may
be thus measured, protracted and computed.
2. The odd measure above Perches is no
valuable here, nor in the former Computations
being always under a square Perch; but in mult
angulars where there be many Remainders, they
must be summed up, and the Perches contained
in them added to the Content before found.
3. This last, and the following Figures (where
I use any Scale at all) are made that they might
not be too large, by a Scale of 400 in an Inch
i. e. by the less Diagonal Scale, each Chain and
Link being counted two.

Fig. 17 III. If this multangular Figure be conceived
to represent a Close of seven sides, which is t
be measured, I begin at the remarkable Angle A
and going round the Close (with the Sun)
find the sides to be in measure 3.11, 2.49, 2.47
1.77, 4.11, 2.29, 4.37. Then

Then I measure the four Diagonals, BD, DF, FB, and BG, in the order that I named them, and I find them to be 3.45,4.97,4.13, and 4.36, which is as short a way as can be taken, to prevent unnecessary Walks. *Fi.* 17.

But when I come to protract by the help of my Scale and Compasses, I first make the Trangle BCD of the first Diagonal, and the second and third sides. Then the Triangle D E F upon the second Diagonal, and fourth and fifth sides, and upon the same Diagonal as a common Base, the Triangle B D F of the first, second and third Diagonals.

Next of the same third Diagonal, together with the fourth and sixth sides I make the Triangle B F G, and upon the fourth Diagonal as upon a common Base with the first and last sides the Triangle A B G, so is the whole Close plotted.

And now it stands visibly reduced into two Trapezio's ABFG and BDEF, together with the Triangle BCD, which I shall not now cast up, having so often shewed how such work is to be done.

But I must acknowledge that this sort of plotting of parcels of Land that have many Angles, requires not only more care and pains, but better skill and memory than to draw Diagonals upon Paper, when the Plot is already taken by the plain Table, or other standing Instrument. I shall therefore to help my young Practitioner in this case, advertize him of two easie ways to help himself, so as to be out of danger of mistakes.

One

Fi. 17 One way is to divide the Multangular Field into two or more parts as the laſt might have been by the Diagonal BF; then might each parts have been meaſured ſeverally, asif they had been ſeparated by a Pale, or were ſundry Mens Lands parted by a Boundary.

Another way that much helps both the underſtanding and memory, is to draw a Rude Draught of the Figure of the Land you intend to meaſure, not only as to the ſides, but alſo neceſſary Diagonals. Then meaſuring the Lines upon the Ground correſpondent to thoſe on the Paper, (which by the help of the Draught may be eaſily hit) ſet the Lines as you meaſure them upon the Lines of the Draught, as if it were the true ones, and when you have finiſhed your meaſuring, protract it truly. Such as you ſee here (but it's better larger) will do your buſineſs, for 'tis not a Pin matter how rude or falſe the Lines or Angles be, reſemblance being all that is deſired.

CHAP. VII.

Concerning the meaſuring of Circles, and their Parts.

I Have hitherto abſtained of purpoſe from meddling with the Circle and its Parts, that I might lay thoſe things cloſe together without unneceſſary mixtures, that are of the greateſt uſe.

'Tis

'Tis wonderful rare, if a Land-meeter ever have occasion to measure any Field or parcel of Land, that will prove either Circle, Semicircle Quadrant, or Sector. Sometimes indeed there will be a little crook in an old Hedge bowing like an Arch: But I have never seen any offer to measure it as a Segment, but always take it as an Angle or Angles.

Yet because it may be expected I should say somewhat of those things, I shall briefly do it.

I. To measure a Circle in the more exact way is to square the Diameter, and to multiply that Square by .7854, so shall the Content be in Integers and Decimals.

But the more usual and quick way (and near enough for any use we shall make of it) is to multiply the half of the Periphery or Circumference by the Semidiameter.

In like manner to find the Content of a Semicircle, Quadrant, or Sector made up of Semidiameters, and arched Lines, multiplying the half Arch by the Semidiameter, gives the Content.

But that which falls out most frequently in Mensuration (though seldom much regarded save where a curious exactness is required) is that particular sort of Segment, which we call a *Section*, less than a Semicircle, such as this Figure ABC. And to find the Content of it, the center of the Circle, whereof this is a Section must be first found out, as here at O, from which Lines drawn to A and B, make up the Sector AOBC; which being measured according to the last Rule, and from the Content *Fig.* 19.

Fig. 19. thereof, the Content of the Triangle A O B ſubſtracted, the difference or reſidue is the Content of the Section A B C.

But two Queſtions may be here demanded: 1. How may the Center be found? 2. How may ſuch a portion of Land be truly protracted and computed?

To the firſt I anſwer, that the moſt exact and artificial way is by making a Mark any where in the Arch.

As for Example.

At the Point C; and then (by a Problem known not only to every Surveyor, but to ordinary Carpenters and Joyners, for finding the center of a Circle, whoſe Circumference will paſs through 3 given points that are not in a right line as ACB) to find the Center O. But if you know not how to do it ſo, croſs the Line A B in the middle, as here it is done by the Perpendicular OC, ſo you may by a few trials find both the due extent of your Compaſſes, and the point in the Perpendicular that will fit your purpoſe near enough; for if a little error be committed in making up the Sector, the moſt of it goes off again in the ſubſtraction of the Triangle.

II. For the latter you may take this ready courſe: Meaſure the length of both your Lines, (the Chord and the Arch) and their diſtance at the middle of them both. Then when you come to protract, firſt take the length of your right Line from the Scale, and having laid it down, croſs it in the middle at right Angles with a dry Line as in the laſt Figure, ſo ſhall it inter-

interſect the Line A B in the point E; then from the ſame Scale take the meaſured diſtance between the two lines in the middle, and ſet it upon that dry line from the Interſection at E; to the point C. Then by trials find a due place in the dry line O E C, and ſuch a diſtance with your Compaſſes, that the one Foot reſting in that line, the other may deſcribe the Arch A C B, and the Section is protracted. *Fig.* 19.

Theſe few hints are as much as I thought neceſſary for my Country Practitioner concerning circular lines; but if he think otherwiſe, there are large Treatiſes enough, and particularly thoſe I mentioned, whoſe Rules (though ingenious, ſound, and fit to be known by every one that intends to plunge deep into Mathematical Studies) will not (I think) be of that uſe to him in ordinary meaſuring, that I ſhould tranſcend the intended bounds of brevity to tranſcribe them.

CHAP. VIII.

Concerning customary Measure, and how it may be reduced to Statute-Measure, & e contra, *either by the Rule of Three, or a more compendious way by Multiplication only.*

WHereas the Statute-Perch or Pole is 16 Feet and a half, and no more, there be Poles of larger measure used in many places, as of 18, 20, 21, 24, and 28 Feet, yea in some 22 Feet and a half. It were therefore very convenient, that our young Surveyor were furnished with a Chain fitted to the customary measure of the Country where he lives, as I use to make Chains for my self and Scholars of 21 or 24 Feet to the Pole for *Lancashire* and *Cheshire*, where those Measures most obtain. But because these are too large and cumbersome for small Closes, it is very convenient, instead of one Chain of 100 Links, to make two of 2 Poles apiece, each Pole divided into 25 Links as that of 100 is, which two half Chains may in measuring large Fields be tied together by the Loops with Pack-thread, or joyned by a buttoning Key-ring for more speedy dispatch; but in smaller we may use the half Chain of 50 Links, only taking care that we count not half Chains for whole ones.

And

And in theſe caſes where the Poles are large and the Cloſes ſmall, it were ſtill more convenient if you had a Chain of 2 Poles only, divided into 100 Links. Only you muſt then take notice, that whereas working by whole Chains and Links, the firſt Multiplication, after five cut off, gives the Content in Acres and Parts. The like work by half Chains and half Links will give the Content in Roods or quarters of Acres, and parts of ſuch Roods.

But though it is no hard matter (for one that can find out the length of a Link by dividing the number of Feet in a Chain by 100, and provide himſelf of good Iron-wyre, and Curtain-Rings to make it of, and a ſharp edged File, and round noſed Plyers to make it with) to be furniſhed with ſuch a Chain; yet becauſe every one cannot do this, I ſhall ſhew you how you may eaſily and yet very truly reduce Statute-meaſure into cuſtomary, that ſo the Chain before deſcribed may do your buſineſs all *England* over.

Know therefore (for a ground to go upon) that Acres bear proportion to one another, as the ſquare of their Poles; and therefore if you multiply 33, the number of half Feet in the Statute-pole by it ſelf, which gives 1089, and alſo multiply the number of half feet contain'd in a Pole of that meaſure you would reduce into; in the ſame manner you may by the Rule of Three reverſe obtain your deſire, making to that purpoſe 1089 the firſt number, the Statute-meaſure the ſecond, and the ſquared half-feet of the Pole given the third. As for example:

Sup-

Suppoſe of a Cloſe meaſured by the Statute-Pole, the length and breadth, and their product be as here repreſented in the Margin. And it is deſired that the Content may be caſt up according to our large *Cheſhire* meaſure of eight yards or 24 feet to the Pole or Rood (as we call it:) Then before I cut off any Figures, I conſider that in the Statute-pole are 33 half-feet, and in the *Cheſhire*-pole 48, then multiplying 33 by 33, and 48 by 48; I have theſe two ſquare numbers, 1089 and 2304, which together with the ſaid product may be thus placed. 1089. 672693 :: 2304, and ſo multiplying 672693 by 1089, and dividing their product being 732562677 by 2304, the quotient is 317952, from which if 5 figures toward the right hand be cut off, and dealt withal as was taught in the fifth Chapter, the Content by our cuſtomary meaſure of 24 feet to the Pole, will be 3 acres, 0 roods, 28 perches as here appears:

9 33 L.	
7.21 B.	
933	
1866	
6531	
672693 prod.	

3 | 117952
4
---
1 | 71808
40
---
28 | 72320

But

But if the lines on the land had been meaſured according to our cuſtom here, of 24 feet to the pole, and the Content muſt have been found according to Statute-meaſure: then I muſt have multiplied the product by 2304, and have divided that latter product by 1089. And in the ſame method you may proceed in all or any of the reſt. But the truth is, that though this way be very exact, plain and comprehenſive, ſuiting all the cuſtomary meaſures before-mentioned without fractions, which for my Learners ſake I ſtudiouſly avoid, and for that reaſon reduced my poles to half-feet, it is ſomething tedious, except he knows how to relieve himſelf by a large Table of Logarithms, or at leaſt a ſet of *Nepair*'s-bones, which I cannot ſtand here to treat of. Therefore to contract the work a little, take notice, that all the cuſtomary poles before mentioned, (ſaving only thoſe of 20 and 28 feet, which I ſuppoſe are ſomewhat rarely uſed, becauſe I never heard nor read of them (to my remembrance) ſave only in Mr *Holwell*; all the reſt I ſay, are capable of being divided into half-yards: And therefore if inſtead of ſquaring the half-feet you ſquare the half-yards of both poles, and work with them, you will attain the ſame end without any regardable difference, the ſmall diverſity that there is being uſually in the uſeleſs remainders, not at all affecting the deſired Quotient that gives the anſwer near enough for uſe.

11
11

As for Example.

11
11
121

16
16

96
16

256

If I had squared 11, the number of half-yards in the Statute-pole, which would make 121, and also 16 the number of half-yards, in our *Cheshire*-pole, which would make 256, as appears in the Margin, and then multiplied the first product 672693 by 121, the second product would have been 81319853, which being divided by 256, the Quotient would have been (as before) 317952. And this way is in a manner coincident with Mr *Holwell*'s first method.

Take notice also further once for all, that whether you use either of these or the following Methods, you need not reduce the particular Squares, Triangles, or Trapezia's severally; but sum up all their products together, and then reduce all at once.

But if you would reduce Statute-measure into Customary by Multiplication only, take notice of this present Table following.

The Content by the Statute-pole being multiplied by	.84027 .68062 .61734 .53777 .47265 .34725	Gives the Content by the Pole of	18 20 21½ 22 24 28	Feet.

The

The uſe of this Table.

When you have multiplied Lengths by Breadths, or Baſes by half perpendiculars, multiply theſe Products by the Decimal Fractions anſwering to the Cuſtomary-meaſure, into which you would reduce Statute-meaſure, and from that latter product, firſt cut off five places towards the right hand as not to be regarded, (being only parts of a ſquare Link) then cutting off 5 more, and proceeding to multiply by 4, and then by 40, as hath been often ſhewed, you will have the Content by that Cuſtomary-meaſure.

Example.

Suppoſe once more the length of a Cloſe meaſured by *Gunter's* Chain, and multiplied by the breadth meaſured alſo by the ſame, produced 672693 ſquare Links; and it is deſired that the Content may be given in *Cheſhire* meaſure of 24 to the Pole: You muſt multiply 672693 by .47265, the decimal fraction anſwering to 24 feet, and from that product being 317948 34645 cut off and caſt away 5 places, and the reſt being 317948, are in the uſual way eaſily reducible into 3 Acres, 0 Roods, 28 Perches, as here appears, agreeable to what it amounted to in the former Method.

317948	
	4
171792	
	40
28171680	

But

But if you measured by a Chain of Customary Poles, and desire to know what the Content is in Statute-measure; this following Table is for your purpose.

	Feet.			
The Content measured by the Pole of	18 20 21 22½ 24 26	being multiplied by	1.19008 1.46923 1.61983 1.85950 2.11570 2.87970	gives the Content by the Statute-pole.

To understand which, take this Example.

Suppose the length and breadth of a long Square being measured by a Chain of 24 feet to the Pole, and multiplied together, make their product 317952, let this be multiplied by 2.11570, which answereth to 24 feet, and the latter product will be 67269104640, from which if you cut off and cast away 5 places towards the right hand, the remainder is 672691, which in the usual way is easily reduced to 6 Acres, 2 Roods, and 36 Perches, as you see here.

```
6[72691
      4
2190764
     40
36[30560
```

One thing more and I have done with this business of Reduction. If the Content to be reduced, be given cast up into Acres, Roods, and Perches, reduce all into Perches, and then in other respects work as before, either by the Rule of Three, or by this last Method of Multiplication

plication only. So ſhall you have the Content in ſquare Perches according to the Meaſure deſired, which you may reduce into Acres by dividing them by 160, and if any thing remain, that remainder being divided by 40, will give you the Roods in the Quotient, and the latter remainder the number of ſquare Perches.

For tryal of which Rules mind the Anſwer of theſe two following Queſtions wrought all three ways.

Queſt. 1. *How many Acres, Roods and Perches, according to the Pole of* 18 *Feet, are contained in* 5 *Acres,* 3 *Roods, and* 11 *Perches, Statute-meaſure?*

Anſw. 4 Acres, 3 Roods, and 22 Perches, as here apppears:

I. Method.

		A.	R.	P.
33	36	5	3	11
33	36	4		
99	216	23		
99	108	40		
1089	1296	931		

1089. 931 :: 1296

1089

8379
7448
9310
1013859

1296)

I

296) 1013859 (782
9072

10665
10368

2979
2592

387

160) 982 (4
640

40) 142 (3
120

22

II. Method.

11 × 11 = 121

12 × 12 = 144

121. 931 :: 144

121

931
1862
931

144) 112651 (782
1008

1185
1152

331
288

43

16.)

160) 782 (4
640
40) 142 (3
120
22

Or thus:

40) 782 (19
40
382
360
22

4) 19 (4
16
3

III. Method.

.84027
931
84027
252081
756243
782.291 37

160) 782 (4
640
40) 142 (3
120
22

Queſt. 2. *How many Acres, Roods and Perches, of Statute-meaſure are contained in* 8 *Acres,* 3 *Roods (or Quarters) and* 21 *Perches of* 21 *Feet to the Pole?*

Anſw. 14 Acres, 1 Rood, and 21 Perches, as appears by the three following works in the ſeveral Methods.

I. Method.

A	R	P		
8	: 3 :	21	42	33
4			42	33
35			84	99
40			168	99
1421			1764	1089

1764 . 1421 :: 1089
1764

5684
8526
9947
1421

1089) 2506644 (2301
2178

3286
3267

1944
1089

855

160) 2301 (14
160
701
640
40) 61 (1
40
21

II. Method.

14 11 196 . 1421 :: 121
14 11 196
56 11 8526
54 11 12789
196 121 1421
121) 278516 (2301
242
365
363
216
121
160) 2301 (14 95
160
701
640
61

40)

40) 61 (1
40
21

III. Method.

1.61983
1421

161983
323966
647932
161983

2301.77843

160) 2301 (14
160
701
640
40) 61 (1
40
21

CHAP.

CHAP. IX.

How a man may become a ready Measurer by Practice in his private Study, without any ones assistance or observation, till he design to practice abroad.

THis Art above all parts of the Mathematicks, is burdened with two Inconveniencies to the young Practitioner: The necessity of having one to assist him in measuring his Lines with the Chain, as oft as he would practice his skill, or get more, and the exposing of his unreadiness to the view of middlesome people, while he is yet raw and unexperienced, as every one must needs be at first: Both which may in a good measure be avoided by this easie knack.

Take a small Packthread, and by knots about half Inch asunder divide it into an 100 parts, as *Gunter*'s Chain is divided; So shall these small divisions between the knots answer to Links; and if they be not exactly of one length, the matter is not very weighty, but the more equal and short they are, the better. Having this String thus prepared, and marked with longer and shorter pieces of thread tied in the knots, so as you may readily see where is the middle, and where your divisions of 25 Links, and the smaller divisions of 10 links begin and end; make all sorts of Figures in your Study or Chamber, by marking places, or sticking Knives

or Bodkins at pleasure for Angles, accounting the straight lines betwixt them for sides, and so measure the Figures by your knotted string, and cast up the Content by a Scale. This Work you may manage with your own hands in private, and so make your self very quick and ready when you begin to measure for good and all, as I once made a full experiment, and it was thus:

When I first began to instruct Youths in Mathematical Learning in *Warrington*, some of my Boys Parents desired a sensible demonstration of their Sons proficiency in somewhat that they themselves could in some measure understand; and particularly pitched upon measuring a piece of Land: Whereupon I took four or five of my Scholars to the Heath with me, that had only been exercised within the Walls of the School. and never saw (that I know of) so much as a Chain laid on the ground: and to the admiration of the Spectators, and especially of a skilful Surveyor then living in the Town, they went about their work as regularly, and dispatched it with as much expedition and exactness, as if they had been old Land-meters.

CHAP. X.

How to measure a peice of Land with any Chain of what length soever and howsoever divided; yea with a Cord or Cart-Rope, being a good Expedient when Instruments are not at hand of a more Artificial make.

IF you can procure a Chain, and find it is not divided as before hath been shewed, but into Feet or quarters of Yards, or any such vulgar divisions, make no reckoning of the divisions at all, but measure it as exactly as you can find out the true length of the whole Chain, and if it fit none of those lengths mentioned in the 8th Chapter, nor any of their halfs, make it to fit by taking off a Link or two, or piecing it out with a string; then dividing the length of that Chain by 100, or the half of it by 50, find the true length of a Link according to our artificial division, and having got a long stick or rod, set as many of those link-lengths upon it as it will hold; Then may you measure all the whole Chains by your regulated Chain, and the odd links of every line by your divided stick or rod, as is manifest in this Example following.

Being far from mine Instrument, and requested by a Friend to measure him a Close, I procure a pair of Compasses, an ordinary Carpenters Rule of two foot, divided into Inches and quar-

quarters, and meeting also with a peice of an old chain seemingly divided into feet, I measure it by the rule, and finding it to be 45 feet long, and some odd measure, I piece it out with a pretty strong Cord that will not stretch much, to 48 feet exactly; then it will serve me for half a Chain of 24 feet to the Pole: This 48 I multiply by 12 (the number of Inches in a Foot) and that product being 576, I divide by 50, the number of links in half a decimal Chain, and the Quotient is 11 $\frac{66}{100}$ Inches, or 11 Inches and an half, and a trifle over: So then dividing a long stick throughout into such parts, each containing 11 Inches and an half, besides the breadth of the nicks, I am provided of Tools to measure Lines to a Link with exactness enough.

In like manner would I proceed with a Cord or Rope, having fitted them to some known length or other: And then for protracting, it were easie with the Compasses to make a plain Scale of a large sort, either upon Paper, or an even piece of Wood; this for once may serve a mans turn well enough.

Besides there is a way of measuring the Perpendiculars of Triangles and Trapezia's upon the ground it self, so as to prevent the necessity of a Scale; for if you have a little Square with a hole in it, to turn upon the head of a little stick, which you may fix where you please, as you are measuring the Base of a Triangle, or the Diagonal of a Trapezium, you may by a very few trials find the place where the one Leg will be just in the Line which you are measuring, and the other point at the Angle from which the

Per-

Perpendicular falls on it, and then the space between your Stick and that Angle truly measured, is the Perpendicular.

If you have not such a Square, a square Trencher, or any end of a Board that hath one right Angle, and two true sided, will supply the want of it.

And now that I am mentioning this way of measuring, I shall make bold to add, that this is a good way (and as such ordinarily used by that general Scholar and reverend Minister Mr *Samuel Langley* of *Tamworth* in *Staffordshire*, whose ancient acquaintance I have long esteemed both mine happiness and honour) to measure a Trapezium thus, though it be protracted afterwards; for by measuring the Perpendiculars as aforesaid, and observing at how many Chains and Links end the said Perpendiculars meet the common Base, the whole Trapezium may be truly protracted, without going about it; this little Square competently supplying the place of an Instrument, which is usually called a Cross or Square, made up (as it were) of two small Indices, like those for a Plain-Table (but much less) with fore-sights and back-sights, and cutting one another at right Angles, put together, and having an hole at the Center, like those things which here in *Cheshire* we call *Yarndles*, being used by Country Housewives in winding of their Yarn.

CHAP.

CHAP. XI.

Concerning dividing of Land Artificially and Mechanically.

WEre it suitable to my Design or Humour to be copious or curious, I had here a fair opportunity; for four or five modern Survey-Books of the best accompt lying open before me, would tempt me to transcribe abundance of ingenious things; but for reasons often hinted before, I shall confine my self to a few plain things that will competently do this business.

1. To divide a Triangle into any parts required; divide the Base as the Demand imports, then shall Lines drawn from the Points of Division to the opposite Angle finish the Division of the Triangle.

Example.

Fig. 20. AC, the Base of the Triangle ABC, being divided into 12 equal parts, a Line drawn from the Angular point B to the point 6 divides the Triangle into two equal parts 2 Lines drawn to 4 and 8 divide it into three equal parts. 2 Lines drawn to the Points noted with 3, 6, and 9, divide it into four equal parts; and so lines drawn to 2, 4, 6, 8, 10, divide it into six equal parts.

Also it is very obvious, that if the same Triangle were so to be divided, that the one part should be double to the other; a Line drawn from

from B to 4 or 8, doth the work. Or if it be required to divide it into two parts, ſo as the one ſhall be triple to the other, a Line drawn from B to 3 or 9, compleats the Work. So alſo a Line from B to 2 or 10 divides it into two parts, whereof the one is quintuple (or five-fold) to the other, and a Line from B to 1 or 11, divides it into two parts, whereof the one is 11 times as large as the other. *Fig.* 20.

Further yet, If it were required this Triangle ſhould be ſo divided, that the two parts ſhould in quantity bear proportion, as 5 and 7, a Line from B to 5 or 7, doth that feat.

But to deal plainly with you, I muſt confeſs that ſometimes the Diviſion will be a little more intricate than thus, yet not ſuch, but that the ſeeming difficulty may be eaſily overcome, by obſerving the method wherein I ſhall ſatisfie the following demand.

Suppoſe a large Triangle of common Land be to be divided amongſt three Tenants A, B, and C, according to the quantity of their Tenements, A having 19 Acres of Land to his Tenement, B 13, and C 7, the Baſe of the Triangle being found by meaſure to be 17 Chains and 27 Links; and the Demand is, where the point of Diviſion muſt be placed in the Baſe, ſo as Lines drawn from thence to the oppoſite Angle, ſhall truly limit each mans part?

To anſwer this let us add 13 and 07 to 19, (as in the margin) and they give 39: So is the work plainly reduced to the Rule of fellowſhip; and therefore to find every mans diſtinct portion, we need only to multiply the Baſe by his par-

19
13
07
39

Fig. 20. particular number, and divide that product by 39, the sum of all their numbers as here is plain:

A

39 . 17 . 27 :: 19

19

15543

172

39) 32813 (841 14/39

312

161

156

53

39

14

B

39 . 17 . 27 :: 13

13

5181

1727

39) 22451 (575 26/39

195

295

273

221

195

26

C

39 . 17 . 27 :: 7

7

39) 12089 (309 38/39

117

389

351

38

From

From these Operations it is plain, that if we set off from the Angular point where the Base begins, 8 Chains 41 Links, and a little above the third part of a Link upon the Base for A, and where that ends, 5 Chains and 75 Links and $\frac{2}{3}$ of a Link for B, and consequently leave between this second division and the other end of the Base 3 Chains and almost 10 Links for C; Lines drawn from those points of division to the opposite Angle, will give each man his due. *Fig.* 20.

What I have said touching the division of Trangles upon their Bases, will with a little variation serve for the dividing of all sorts of Parallellograms, whether Squares, Long-squares, Rhombus's, or Rhomboides's: all the difference is, that instead of drawing Lines from Points in the Base to the opposite Angle, you must draw parallel Lines from Points in one opposite side to another, as will be sufficiently plain by this one Instance.

Suppose the square Figure in the 8th Prop. of the second Chapter to represent a Close of six Acres, and I am to cut off an Acre at the side AC, having set off the 6th part of the Line CD, from C towards D, and also from A towards N, a Parallel drawn between those Points, takes off exactly a 6th part or an Acre. *Fig.* 7.

If it be not thought convenient (as in some cases it is not) to cut off a piece so long and narrow, you may by the Rule of Three find what other length of any greater breadth will limit an equal quantity to it. Or you may multiply the breadth by 2, 3, or any other, and divide the length by the same number that you

Fig. 7. multiplied the breadth by. Or lastly, if you set out a double proportion that is $\frac{2}{6}$ or $\frac{1}{3}$, from C towards D, and from the Point where it falleth, draw a Line to the Angle A, you will have a Triangle equal to $\frac{1}{6}$ of the Square A C D N.

But to return to Triangles, (the most simple and primitive of all Rectilinears, and therefore the most considerable in this case of partition, as giving Laws often to the rest:) It may fall out that a Triangle must be divided (convenience so requiring) by a Line from some Point in a side, so as that Line may either be parallel to some other side, or not parallel to any. For the former case take this Example following out of Mr *Wing. Lib.* 5. *Prob.* 5.

Fig. 21. Let A B C be a Triangle given, and it is required to cut off $\frac{3}{5}$ by a Line parallel to A B. First, on the Line AC, describe the Semi-circle, A E C, whose Diameter C A divide into five equal parts according to the greater term, and upon three of those parts (the lesser term) erect the Perpendicular D E, which cutteth the arch Line in E; then set the Line C E from C to F, and from thence draw the Line E G parallel to A B; so will the Triangle C G F contain $\frac{3}{5}$ of the Triangle A B C, as was required.

Fig. 22. Now for the latter case, when the line of partition goes not parallel with any side, take this Example:

Let A B C be a Triangle given to be divided into two parts which shall bear proportion

tion to one another, as 3 and 2, by a Line drawn from the point D in the Base, or Line A C. *Fig.* 22.

From the limited point D, draw a Line to the Angle B; then divide the Base A C into five equal parts, and from the third point of Division draw the Line to E, parallel to BD. Lastly, from E draw the Line E D. So shall the Trapezium ABED be in content, as 3 to 2, to the new Triangle D E C.

I have now done with the Division of Triangles, when I have added these three Advertisements.

1. You must be sure to take very exactly the distance of every point, where a dividing line cutteth any side, to one of the ends of the same side, as in this last Figure, the distances BE and A D, which distances being applied to the Scale by which the Triangle was protracted, will shew at how many Chains and Links-end you are to make your dividing Line on the Field it self.

2 The proportions by which you are to divide, are not always so formally given as in the former examples, but are sometimes to be found out by Arithmetical working, as in this case.

Suppose a Triangular Field of 6 Acres, 2 Roods, and 31 Perches, must be divided so as the one of the two parts shall be 4 Acres, 3 Roods, and 5 Perches, and the other (consequently) 1 Acre, 3 Roods, and 26 Perches, reduce both measures into Perches, and the one will be 765, and the other 306. Their Sum is 1071, which by their common measure being

Fig. 22. reduced into their loweſt terms of proportion in whole Numbers, will be 5, 2, and 7, which ſhews that the Triangle being divided into 7 equal parts, the one muſt have 5 of thoſe 7 parts and the other 2. And obſerve, that it will be ſufficient to find the common meaſure between the Sum of the terms and either of the terms; the method whereof is ſhewed in every Arithmetick Book for reducing Fractions in to their loweſt terms.

But if my unlearned Reader cannot skill of that work, he may multiply either of the parts; (as ſuppoſe 765) by the length of the Baſe, which we will ſuppoſe to be 8 Chains and 75 Links, or 875 Links: and that product divided by 1071, (the Content of the whole Cloſe in Perches) gives by the Rule of Three direct, 625 Links, or 6 Chains and 1 Pole, the true diſtance from either end of the Baſe, that his mind or occaſions may direct him to begin with, to the point of Diviſion; for the Diviſion muſt be not only for proportion or quantity, but alſo as to poſition or ſituation of parts upon the Paper, as it is required to be on the Ground.

3. In theſe and all other diviſions of Land, where a ſtrict proportion in quantity is to be obſerved, you muſt have reſpect to the Rules hereafter following, concerning meaſuring of uneven Ground, *Chapter* 15. eſpecially if one part prove much more uneven than another: and if there be any uſeful Pond, or Well to draw your Line of Diviſion through it; but if it be an unuſeful Pond, Lake or Pudddle; or if there be any boggy or barren ground, that muſt be caſt out

out in the diviſions; meaſure that firſt, and ſubſtract it from the Content of the whole Cloſe, and then lay the juſt proportion of the remainder on that ſide that is free from it, that the other may have its juſt part alſo, beſides that which is uſeleſs.

What hath been ſaid, with an ordinary meaſure of diſcretion, may ſufficiently inſtruct a young Artiſt to divide Triangles, Parallellograms, and regular Polygonials, in an artificial way: but becauſe many Cloſes and open Grounds are Trapezia's, and many irregular Polygons, and even thoſe that are regular enough, may fall under an Irregular diviſion, in regard of the quality of the Land, Woods upon it, or Quarries in it: Or the conveniences of ways, Currents of Water, ſituation in reſpect of adjacent Lands, &c. I ſhall propoſe a Method, which though it hath ſomewhat of the Mechanick in it, will be ſingularly uſeful in ſuch caſes.

Let A B C D be a Trapezium to be divided betwixt a young Heir and his Mother, ſo as his part may be double to hers. Having by the Diagonal B C divided it into two Triangles, I find the Content of the Triangle A B C to be 138550 ſquare Links, and the Triangle B C D to contain 103468, in all 242018 ſquare Links, which if reduced, as hath been formerly taught, would amount to 2 Acres, 1 Rood, and 27 Perches, (for which ſee *Chap.* 5.) *Fig.* 23.

But as to my preſent work, they are in a better Order already. Dividing then 242018 into three parts, each of them is 80676 and $\frac{2}{3}$,

Fig. 23 two therefore of those third parts must contain 161353 and $\frac{1}{3}$, which $\frac{1}{3}$ being inconsiderable, I regard not.

Then resolving to lay out the double part towards the Line BD, I strike at adventures the Line E F, and measuring the Trapezium bounded by that Line, and the opposite side B D, together with the Interjacent parts of the Lines A B and C D, I find it to contain 119140 square Links, but because it should have been 161353, I substract 119140, out of 161353, and their difference is 42213, and perceiving that the Lines A B and C D are very near parallel, and finding their distance where they are cut by the Line E F, to be 326 Links, or 3 Chains and 26 Links, I divide 42213 by 326, and the quotient is 129 Links and almost half, at which distance I draw the Line G H parallel to E F; so shall the Trapezium G B D H be the Heirs part.

Another way whereby this may be preformed, is thus: Finding the Triangle A B C, to contain 138650 square Links, substract it out of the Heirs part, *viz.* 161353, the difference 22803, shews how many square Links must be taken out of the Triangle B C D, and added to the Triangle A B C; Which to perform with all necessary exactness; suppose the side or line B D to be the Base, which by measure proves to be 344 Links, or 3 Chains, and 44 Links. Say by the Rule of Three direct, If the whole Content of the lesser Triangle, *viz.* 103468, give 344; what shall 22803 give? so will the result be 75 links, and somewhat more than $\frac{4}{5}$ of a Link;

a Link, for 22803 multiplied by 344, gives 7844232, which being divided by 103468, the Quotient is $75\ ^{1\,8\,4\,1\,3\,2}_{0\,3\,4\,6\,8}$; or (according to decimal Diviſion) 75.8131, which is (as I ſaid before) ſomewhat more than 75 Links and $\frac{4}{5}$, wherefore extending your Compaſſes upon the Scale to almoſt 76 Links, ſet that diſtance upon the Line BD, from B to I, and draw the Line CI: ſo ſhall the Trapezium, ABIC be double to the Triangle ICD, within ſo ſmall a matter as is not worth regarding, though the Land were a rich Meadow. *Fig.* 23.

I hope I need not ſtand to tell any man of ſenſe, that if he pleaſe he may begin with the leſs part, and take out that: or if there be many Partners, he may divide betwixt any one and all the reſt, (putting their parts together) and then by the ſame method ſubdivide amongſt them till each hath his due ſhare; nor to ſpend many words in telling him he muſt ſubſtract where he hath by a ſeparating Line at adventures, or by chooſing out a Triangle, taken too much; as I added, when I took too little. Nor laſtly, that theſe methods are not only applicable to Trapezia's, but to any triangular Figure whatſoever, whether regular or irregular.

CHHP.

CHAP. XII.

Concerning the Boundaries of Land, where the Lines to be meaſured muſt begin and end.

IF there be no agreement between the Parties concerned, (for if there be, that muſt be obſerved) Reaſon and Cuſtom are the Surveyors Guide.

The Farmer ſpeaks loudly, that when a piece of Arable or Meadow-land is let for a year to be ſown or mown, no more ſhould be meaſured nor expected to be paid for, either to the Letter or Workmen, than the Plow or Sythe can go over. So alſo when a parcel of Land is let for Paſture by meaſure to a Farmer, it ſeems very reaſonable, that all and only ſo much ſhould be meaſured as is uſeful to that purpoſe.

But Commons to be incloſed are uſually meaſured (except it be otherwiſe agreed) to the uttermoſt bounds of every mans particular proportion, without any allowance for Ditch or Fence; every man being to make them upon his own of what breadth he pleaſes: Nor is this unreaſonable, for 'tis as good for one as another, and the rate paid to the Lord is uſually very little, ſometimes nothing.

It is alſo very uſual in meaſuring betwixt Lord and Tenant, in caſe of Leaſes for Lives, and

and long terms of years, to extend the Lines to the utmoſt bounds of the Tenants claim, taking in the very Walls, Hedges, and Ditches: but this is accounted very hard, and oft proves very unequal among the Tenants of a Lordſhip; ſome being forced to make much more waſte of their Ground this way, than others that hold as much or more. But where the Cuſtom obtains the Surveyor muſt obſerve it: For it is others work to appoint what muſt be meaſured, and his only to meaſure truly what is ſo appointed. A good Landlord may (and will be apt) to conſider it in his Rates, and a bad one 'tis like will be tenacious of a Cuſtom to his own advantage.

Laſtly, In caſe of Sale by meaſure at a rate agreed upon *per* Acre (no Boundaries being ſpecified in the Bargain) the Rule (as I had it from an old famous Surveyor many years ago) is to extend the Lines to the quick Wo[illegible]row, that is, as Reaſon prompts me to underſtand him, to the place where the quick-wood actually groweth or where according to cuſtom it ought to be ſet.

I confeſs theſe things are trivial, but yet more neceſſary to be known, than many artificial things much ſtood upon, for a young Artiſt, whom ignorance of theſe things may expoſe to conſiderable miſtakes in practice.

CHAP.

CHAP. XIII.

Containing a description of the Plain-Table, the Protractor, and Lines of Chords.

THough what hath been already said, may competently suffice to instruct my young Artist in measuring a Close of Land, yet to advance him a degree higher in useful knowledge, I shall take occasion to describe unto him the Plain-Table, which with Mr *Wing* I account the best of all fixed Instruments: This Instrument consists of several parts.

1. The *Table* it self, which is a Parallellogram of Wood fourteen Inches and a half long, and eleven Inches broad, or there abouts, and for necessity may be made by an ordinary Countrywork man of one Board: but for neatness, convenience of carriage, and freedom from warping, it is usually made of three little Boads joyned together side ways, with a ledge at each end to hold them fast together, and upon the middle Board a Socket of Brass fixed with three Screws, and with a fourth to be fastened on the head of a three legged Staff; of which anon.

2. A Frame of Wood fixed to it, so as a sheet of Paper being laid on the Table, the Frame being forced down upon it, squeezeth in all the edges, and makes it lie firm and even so as a Plot may be very conveniently drawn upon it: this is usually made with Joynts for more easie carriage, but a plain one may suffice.

Upon

Upon one side of this Frame should be equal divisions, for drawing parallel Lines both longwise and cross-wise (as occasion may require) over your Paper, and on the other side the 360 degrees of a Circle projected from a Center of Brass conveniently placed in the Table.

3. A Box with a Needle and Card, to be fixed with two Screws to the Table, very useful for placing the Instrument in the same Position upon every remove.

4. A three-legged Staff to support it, the Head being made so as to fill the Socket of the Table, yet so as the Table may be easily turned round upon it, when 'tis not fixed by the Screw.

5. An Index, which is a large Ruler of wood (or rather brass) at the least sixteen Inches long, and two Inches broad, and so thick as to make it strong and firm, having a sloped edge, (by which we draw the Lines) called usually, the fiducial edge, and two sights of one height (whereof the one hath a slit above, and a thread below, and the other a thread above and a slit below) so set in the Ruler, as to be perfectly of the same distance from the fiducial edge. Upon this Index 'tis usual to have many Scales of equal parts, and there might be a Diagonal Scale if the Instrument-maker please, and Lines of Chords of sundry lengths: but if you have such a Scale as I before described, you need not to have them here.

The *Protractor* is an Instrument so well known, and so easie to be made and procured, that I shall be very brief in the description of it.

As

As it is usually made, it consists of two parts, a Scale and a Semicircle, but the Scale is no necessary part of it, but serving (if you be not otherwise provided) for other uses before mentioned in the case of Plain Scales.

But the Semicircle is more essential, and it may be made of Brass or other Metal of any convenient size, as four Inches (more or less) for the straight side, this Semicircle being bounded as all others are by two lines, the one right or straight, the other circular.

The right Line is divided in the precise middle by a Point which is in the Center, upon which the Circular boundary is drawn, and two other Arches concentrial with it.

The Center, when the Semicircle goes alone without the Scale should be guarded with two little lips, on each side one, or a little loop, for more convenient turning of the Instrument about upon a Pin fixed in a Paper.

The arched or circular edge is divided into 180 Degrees or equal parts numbred by Tens, upon the upper Concentrick Arch from 0 to 180, and in the lower from 180 to 360. So that by applying the straight edge of the Protractor twice to any Line, (keeping the Center right upon a Pin fixed in the Line) that is with the Semicircle first above it, and then below it, or contrarily, you may draw a whole Circle by the guidance of the Arch, or set out any number of degrees, as will appear more plainly hereafter.

A *Line of Chords* is a Line divided into 90 unequal parts, whereof 60, and the Radius upon which

which the Circle was drawn, are equal, and the Diviſions upon that Line are equal, to the next Extent in a right line, of ſo many Degrees from the beginning of the Quadrant as anſwer thereunto.

When lines of Chords are cut upon wood, 'tis both uſual and neceſſary that there be two Studs of Braſs, the one at the beginning, and the other at 60 Deg. with little holes for the feet of the Compaſſes, when you take the extent of the Radius, to preſerve the line from being wounded by the Compaſſes; and being thus fenced, it will for need do the work of a Protractor, but not altogether ſo commodiouſly.

CHAP. XIV.

How to take the true Plot of a Field by the Plain-Table, upon the Paper that covers it, at one or more Stations.

THere are three ways or methods for doing this work, two more uſual and ordinary, the third more unuſual and extraordinary, though now pretty well known to moſt Surveyors, and in late Books publiſhed. The firſt performs the work by Meaſuring every line from the Inſtrument to every Angle, and is a very ſure ſubſtantial way where it can be done, as it ordinarily may in moſt Cloſes.

The ſecond doth it by meaſuring only the Station or the Diſtances, and is very quick, but not ſo ſure and exact as the other; yet if managed by a skilful Artiſt, that knows how to plant his Inſtrument, ſo as to avoid making acute Angles unneceſſarily, it will come near enough the matter in many caſes; as in meaſuring for Workmen, that take the Mowing or Reaping of Fields by the Acre, or when Tythes are let at a ſmall value *per* Acre, as in poor barren Pariſhes they uſually are.

The third is the way of Circulation or Perambulation, the Inſtrument being oft to be planted, and the Plot to be meaſured about, by which not only difficult Cloſes, but even the thickeſt Woods, yea, Bogs, Meres, and Pools of Water, may be plotted, which by neither of the other methods can be performed.

In all theſe Methods, two things are to be performed:

1. At every Angle where there is no perſpicuous Mark already, as a Tree, Buſh, Stile, *&c.* one muſt be placed, as a white Paper, or ſuch like; or elſe ſome one muſt go from Angle to Angle, and remain there as your Mark to look at, till you bid him remove to another: only when Angles are near you, this labour may be ſpared.

2. When ever you have occaſion to plant your Inſtrument more than once, (as it will often fall out in the firſt Method, and ever in the two latter) you muſt be ſure it ſtand juſt as it did the firſt time for ſituation, for which your Needle if well touched and hung will be good

good direction, but is not thought ſufficient without back ſight and fore ſight, (by ſome practical Surveyors:) I ſhall therefore in due ſeaſon ſhew you that knack. Now for the firſt Method.

I. When you go about to plot your parcel of Land, find ſuch a place in it if poſſible, from whence you can ſee all the Angles, and in that place plant your Inſtrument covered with a ſheet of paper, and turning it about till the Needle playing at liberty, hang over the Flower-de-luce, (or any other notable place that you make choice of (ſcrew it faſt. Then chooſing any convenient place in your Paper for a Center, and to repreſent your ſtation (or place where you fix your Inſtrument) make a prick with the ſmall point of your Compaſſes, to which prick applying the fiducial edge of the Index, (which is eaſily done if you keep the point of the Compaſſes reſting in it) direct the Index, by the ſight to all the Angles, and when through the ſlit or long ſight, you ſee the oppoſite Thread cut the Mark in the Angle, draw a neat dry line along the fiducial edge to or from the Center: then meaſuring from the Inſtrument to every Angle, ſet the meaſure by a Scale and Compaſſes from the Center towards the Angle, upon the line that points at it, making a prick in the line where the Chains and Links reckoned from the Center) do truly end: then ſhall Lines drawn by a ſtraight Rule from prick to prick, give you the perfect plot upon your paper, which you may divide (as hath been before ſhewed) into Trapezia's and Triangles, and

ſo find the true Content. To make which plain, mark this *Example*.

Fig. 24. Suppoſe A B C D E F G H to be a Field; having planted my Plain-Table as before directed at a convenient advantage, ſo as to ſee all the Angles, (as at I) I made a prick to repreſent my ſtation in the little Circle ⊙ marked with I, upon which laying the fiducial edge of my Index, and directing the Sights to all the Angles, I draw dry lines toward A, and all the reſt of the Angles in order from the Center; and then meaſuring upon the ground from the Inſtrument to the Angle A, I find it to be 3.45, which I ſet (by help of my Scale) from the Center to the Point A, and ſo upon all the reſt according to their due meaſures, and then black lines drawn from Point to Point, as from A to B, from B to C, *&c.* limit the true Figure of the Field according to the Scale I uſed, *viz.* of 400 in an Inch.

And now before I paſs to further varieties, let my Reader take notice of theſe following things.

1. From henceforth I ſhall forbear (for brevities ſake) to take any notice of the meaſures of lines meaſured from the Inſtrument to the ſeveral Angles, having ſo often ſhewed how to meaſure by a Scale.

2. When I ſpake of meaſuring from or to the Inſtrument, I always mean from or to that part of ground that is perpendicularly under the head of the Inſtrument, where you are to draw your Plot, which will ever be encloſed with the three Legs of your Staff.

3. That

3. That it's usually the quickest way to measure first from the Instrument to the first Angle, and then back from the second Angle to the Instrument, and so the rest in order, still one from the instrument, and the other to it. *Fig* 24.

4. It is no matter at all whether your Plain-Table be placed towards the middle of a Field, as was represented in this Figure, or at an Angle, as will appear anon.

5. In all workings by this Instrument, you must have a care that the Instrument be not moved out of its due place, till you have finished the work of the present Station, for which purpose, cast your eye now and then upon your Needle, observing whether it continue to hang directly over the same point you set it at when you began your work, and to rectifie your Instrument if you see cause. But because all Tables have not Needles, and where Needles are, they are not accounted over-trusty, make use of the following help.

When you have planted your Instrument, and made a point or prick in your paper, representing your station, set the fiducial edge of your Index to it, and turning it softly about till you find one remarkable thing or other upon one side of the Close, and another on the opposite side as you look through the sights of your Index (which we call fore-sight and back-sight) draw a remarkable Line with Ink, or rather with a Black-lead Pen quite over your Paper, which in this Figure, is represented by the black line K L: and then if you suspect that by any accidental jog, or other casualty, the Instru-

ment is any thing removed, you may eaſily try and rectifie it, by applying the fiducial edge to the ſame Line, and making uſe of fore-ſights and back-ſights again, upon the ſame Marks which you before obſerved upon the oppoſite ſides of the Cloſe.

But if there be no convenient place for the placing of your Inſtrument, whence you may ſee all the Angles of the Field, more ſtations muſt be made uſe of thus:

Fig. 25. Let A B C D E F G H I K L M N be a Field whoſe Angles cannot be all ſeen from any one Angle, or other place in it: I plant my Inſtrument at the Angle A, and if it have a Needle I mark what Degree of the Chord it cuts, or turn about the Table on the head of the Staff, till the Needle hang over ſome remarkable place; (as ſuppoſe the Flower-de-luce) and ſcrew it faſt; then ſetting up a Stick with a white paper or cloath on the head of it, where I intend my ſecond Station (as here at Q.) I made a prick or point in my paper, to ſignifie the point A upon the paper on the Table: to which point I apply the fiducial edge of the Index, and when I ſee the white at Q, ſo as looking through the ſlit, I ſee the thread cut it, I draw the Line O P quite through my Paper with a Black Lead Pen, and then keeping the fiducial edge ſtill upon the ſame point, and turning it round by degrees, I look at the Angles B C D E F L M N, ſtill drawing dry Lines with the points of my Compaſſes, and ſetting on the Meaſures from the ſtation A to every Angle meaſured to or from, as I did in the laſt Example. Then

Then I remove the Inſtrument to the place of my ſecond ſtation, having ſet up a mark at A, and laying the fiducial edge to the Line O P, I turn about the Table upon the head of the Staff, till through the ſlit of the back-ſight, I ſee the thread cutting the mark at A, and then ſcrew it faſt, ſo will my Needle, if a good one, hang directly over the ſame point that it did at the firſt ſtation; but however that be, fore-ſight and back-ſight will do the buſineſs; for which purpoſe it is good to take back-marks as well as fore-marks at every ſtation, as was taught in the Example of a ſingle ſtation, only taking notice that the back-mark when the Inſtrument is planted in an Angle, muſt needs be out of the Field; as ſuppoſe here at O. But to proceed. *Fig.* 25.

Having meaſured the diſtance between the firſt and ſecond ſtation, and finding it to be 7. 10, I ſet it upon the Line O P, from A to Q, where I make another point to repreſent the ſecond ſtation, and turning about my Index with the fiducial edge upon that point, and ſo looking through the ſights at the Angles G H I K, I draw Lines towards them on my Paper, and having meaſured between every one of thoſe four Angles, and the Inſtrument, I ſet thoſe Meaſures as I did the other, with my Scale and Compaſſes, from Q towards every Angle upon his proper Line: and then having drawn the black bounding-lines from A to B, from B to C and ſo round about the Cloſe, the Protraction is finiſhed.

But here to make this Figure yet more ad-

vatageous,

Fig. 25. vantageous, let me (according to my usual method) add some Advertisements.

1. Sometimes a Station is so taken, that you may measure towards two Angles at once, (as here from Q to G and H) in which case you are to set down the Chains and Links where the first Angle falleth, but still be proceeding to the further Angle, causing the remainder of the Chains at the fore-end to advance beyond the former Angle, so going on with whole Chains so far as you can, to which the odd Links at the end are to be added.

2. If at any of your Stations (as suppose A) you can see an Angle (for example E) to which you cannot measure in a direct Line without passing the boundaries of your parcel of Land given to be measured; you may notwithstanding take in that Angle by a strait measured Line, as I have done, provided it may be lawfully done without trespass, and conveniently without troublesom passing of Fences, otherwise it must be taken from another station.

3. I here took one of my Stations at an Angle, and the other within the body of the Field to shew the variety of working taught by other Authors, and that 'tis no great matter where you make your Stations, so you can see the Angles: else it had been full as convenient to have taken my first Station also within the body of the Field, as suppose at R.

4. Though this Figure representeth to your eye only two Stations, A and Q, your fancy may multiply them at pleasure, for suppose the Angle H could not have been seen from A or

A or Q, how eaſie had it been to have ſet up a Mark at S, and then to have removed the Inſtrument thither, obſerving the ſame directions that were given at the removal from A to Q.

II. In the ſecond Method the Inſtrument is to be planted twice, or oftner as occaſion is, the Rules for removal of the Inſtrument fore-ſight and back-ſight, and meaſuring the diſtance of Stations, being the ſame as formerly was taught: but inſtead of meaſuring to and from every Angle, we only view each Angle through the Sights from two Stations, having applied the fiducial edge to the Points repreſenting thoſe Stations, and having drawn Lines with the point of the Compaſſes, or a protracting Needle, the Interſections repreſent the Angles, from which the boundary-lines may be drawn, ſo is the Field protracted. Which that my Reader may underſtand, let him note theſe three Figures.

Here in theſe three Figures the Angles are marked Alphebatically, A B C D E F, *&c.* and the Stations by a ſmall point in a ſmall Circle numbred, 1, 2, or 1, 2, 3, according to their number and order. *Fig.* 26. 27. 28.

The firſt of theſe Figures repreſents the Plotting of a Field at two Stations within it, from both which all the Angles may be ſeen. *Fig.* 26.

The ſecond, performs the ſame work by two Stations taken without the Field, by which Art a Cloſe may be meaſured, though the preſent Poſſeſſor will not give us leave to come into it. *Fig.* 27.

The third, ſhews how the Work may be performed *Fig.* 28.

Fig. 28. formed at three Stations or more, when two ſuch places cannot be found whence to view all the Angles; which laſt having more of difficulty than the two former, (though indeed not very much) and the Explanation of that will ſufficiently help to the underſtanding of them, I ſhall a little explicate the meaning of it in theſe particulars.

1. From the firſt Station taken according to former directions, I ſee the Angles A B C D F G K, and accordingly draw Lines upon my paper towards them from the point repreſenting that Station, by the fiducial edge of my Index with the point of my Compaſſes.

2. Having removed my Inſtrument to the ſecond Station, (and in ſo doing, obſerved the Rules before given, touching ſuch removals) I thence ſee the Angles A B D E F G H I K, and draw lines upon my paper towards them from the point repreſenting the ſecond Station. And now viewing my Work, I find upon my paper Interſections for the Angles A B D F G K, but only ſingle Lines toward the Angles C E H I: therefore,

3. Removing the Inſtrument regularly as before, to a third Station, I thence ſee thoſe four Angles C E H I, and drawing Lines towards them I have interſections for them alſo; ſo that having drawn the Lines A B, B C, *&c.* from one Interſection to another, I have the Field perfectly protracted. For theſe bounding-lines from Angle to Angle, do not only ſignifie the Boundaries of a peice of Land given to be meaſured, limiting the Figure or ſhape thereof

(and

(and are to that purpoſe given in this and all other Survey-books) but alſo are the true diſtance by a Scale from Angle to Angle for the Plot upon the Paper: I mean by the ſame Scale by which the ſtationary diſtances were laid down upon their own lines. And this holds true in all kind of true plotting, whether in this Method or any other. *Fig.* 28.

III. The third Method is that of Circuition, and this hath ſeveral varieties, according to theſe three following Caſes.

1. When the diſtance from Angle to Angle, (without any exception) is meaſured quite round the Plot, either within or without.

2. When the diſtance is taken only between ſome more notable Angles, and the Perpendiculars of the reſt meaſured as you paſs along their Baſes, within the Plot, proper for plain ſolid ground.

3. When the like is done without the Plot, as in caſe of Plotting thick Woods, Meres, Pools, Bogs, *&c.*

The firſt of theſe are very eaſie, conſiſting in nothing but planting the Inſtrument at every Angle (either within or without, as neceſſity and convenience determine it) obſerving the former directions for planting and removing the inſtrument and alſo for meaſuring the ſtationary-lines on the ground, and protracting them on the paper, as is manifeſt in this *Example.*

Let A B C D E F be a Park-pond or Cloſe to be protracted, I firſt plant my Inſtrument at A, and direct the ſights to a Mark in the Angle B, draw- *Fig.* 29.

Fig. 29. B, drawing a dry Line from a convenient Point on my paper towards B on the ground, and having measured by my Chain the distance A B, I set it by a Scale upon the Correspondent Line from A to B, drawing a black-line between them with Ink or a Lead-pen, the Extremities whereof are the Points A and B on my Paper, and the little pricked Line that goes beyond B, represents the remainder of the dry Line drawn at random (as to length) with the point of the Compasses.

Then setting up a Mark at A (if there was none before) I remove the Instrument to B, and laying the fiducial edge to the Line A B, I turn about the Instrument upon the Staff, till through the sight I perceive the thread cutting the Mark at A and my Needle (if I have one) directly over the same point, that it was when it was planted at A, and so screw it fast.

My next work is to lay the fiducial edge to the point B, and direct the sights to C drawing a dry Line towards it, and setting the distance B C measured by the Chain from B to C.

In this manner I proceed, surrounding the Close till I come at last to A, where I began, by planting the Instrument at every Angle, using the help of Back sight and my Needle, as I did at B, and then from the point representing my present station, directing the sight to the next Angle, as I did from B to C.

In the second Case, we do not plant the Instrument at every Angle, but at the more considerable, taking in the smaller by their Perpendiculars from the Base as we pass along, of which

which this following Figure may be an Inſtance.

Let A B C D E F G H I K L be a Cloſe to be meaſured; by planting the Inſtrument only at A C F and K, I have the main ſubſtance of the Cloſe in the Trapezium A C F K, and for the five ſmall Triangles which muſt be added to the Trapezium, they may be eaſily protracted by the help of ſuch a little Square as was mentioned towards the latter end of the *Tenth Chapter*; for thereby finding at how many Chains and Links diſtance from A upon the ground, the Perpendicular B b falleth upon the Line A C; and having meaſured the length of that Perpendicular, and taken in between my Compaſs-points of my Scale, I erect a Perpendicular of that length at b, which is the point upon the Paper, where ſo many Chains and Links determine, as were meaſured upon the Ground, from the Angles A, to the place where the Perpendicular fell on A c, *viz.* at b. Juſt in the ſame manner I raiſe the other Perpendiculars, D d, G g, I i, and L l: and then by the help of the Perpendiculars, I draw (from and to the proper Angles) the Boundary-lines A B, B C, C D, D E, F G, G H, H I, I K, K L, L A; which together with the Line E F between the Angles E and F, give the true Plot of the Field in one large Trapezium, and five ſmall Triangles ready for caſting up. *Fig.* 30.

The third caſe is like the ſecond, that there needeth no new Direction ſo concerning it, but to annex one plain Diagram; all the difference conſiſting in this, that becauſe we cannot go

within it, (being ſuppoſed to be ſome Pool, Bog, or Thicket) we muſt of neceſſity go on the outſide, and conſequently all the Triangles made by inward Angles, and their Lines upon the meaſured Baſes, muſt be excluded by the boundary-lines from being any parts of the Plot, as here is manifeſt.

Fig. 31. Suppoſing A B C D E F G H I K L M N O P, to be a great Pool, though here be fifteen Angles, I plant my Table only five times, *viz.* at A E F H and M, and upon the dry Lines A E, F H, H M, and M A, I raiſe their Perpendiculars in due places, (according to meaſure) and alſo of a right height: By which and my five ſtationary Angles, I draw the bounding-lines of the Plot, excluding all the Triangleſas foreign to it, they being no reſemblances of any part of the Pool, but of Land adjacent.

Where note, 1. That both in this and the former caſe, ſuch a little Square as I mentioned in the *Second Chapter*, will be very uſeful for ſpeedy raiſing of Perpendiculars; but where the Triangles are very ſmall, it need not be uſed, nor the other mentioned *Chap.* 10.

2. That if by reaſon of troubleſome Bruſh-wood, Gorſe, or Bogs, *&c.* I could not have meaſured cloſe to the ſides E F, H I, or L M, it would be the ſame thing if I went parallel to them. And this is a ſhift that the practical Surveyor will oft be put to make uſe of, in other caſes as well as this.

CHAP.

CHAP. XV.

Concerning the Plotting of many Cloſes together, whether the Ground be even or uneven.

THough I deſign not ſo high in this *Manual* as to make my Reader able to Survey Lordſhips and Foreſts, much leſs to draw Maps of Countries, but to meaſure a parcel of Land with truth and judgment; yet I would have him ſo expert, as not to be puzzled, if any ſhould deſire him to draw (as it were) a true Map of a Tenement or ſmall Demeſn, conſiſting of ſeveral Cloſes; for which purpoſe, let him that knows no better obſerve this method.

Suppoſe A B C D E F G H I K L M to be a Tenement or ſmall Demeſn divided into fourteen Cloſes, to be meaſured and protracted according to their ſeveral ſhapes and ſituation. *Fig.* 32. I firſt draw the Plot of the whole by the method of Circulation, planting mine Inſtrument either at every Angle, or only the moſt conſiderable either within or without, as I find moſt convenient. This being done, a Line from B to M gives the Triangle A B M from the firſt Cloſe; In the next place, I go round the ſecond Cloſe beginning at M, then to B, and ſo about (*cum ſole*) to M again: And then for the third Cloſe I plant my Table at C and go round to B, (the Line B C being protracted already) and ſo of all the reſt, ſtill obſerving which are com-

Fig. 32 mon Lines belonging to several Closes (representing the Fences) that I may avoid the trouble of measuring those Lines oftner than once, and lay every part of every Close in its due place; and that I be sure to keep the Instrument throughout the whole Work to its true Position by Needle, fore-sight and back-sight.

There are I confess divers other ways of doing this Work, but none more sure or plain, especially if the ground be uneven; for in that case, if you protract according to the length of Lines measured from your Station to the Angles, you will put your Closes into unproportionable shapes, except you reduce Hypothenusal Lines to Horizontal, by instruments or
Fig. 32. otherwise (which is somewhat troublesome:) and the like may be said when you Plot with the Chain only. Indeed the method of measuring only the stationary distances were very proper for setting out the Figure of each particular Close, provided the distance of the stations be large, and taken (if possible) upon pretty even ground, (which sometimes may be done, though most of the Close be uneven) and the Work so ordered, as not to make too acute Angles: but because this requireth skill and care, I rather advise my young Artist to use the circling way, as ordinarily most commodious in my poor judgment, (but not prejudicing other men's that may differ from me in oppinion) and where need requires let him observe the directions in the 17th *Chapter*.

☞ But which way soever you go to work, there is one very necessary Rule to be observed. If the

the ground being uneven considerably, you must not give up the Content by measuring the Bases and Perpendiculars of the Triangles on the Paper by your Scale; but you must measure the Lines correspondent to them on the Ground, and cast up the Content according to that measure. And if it be desired that you should adjoin to your Plot (as is usual) a Scale of Chains to measure distances by, you must either by making the Forms of Hills erect and reverse, or some other Note in writing, mark out your uneven ground, lest those that try it by the Scale, judge your work erroneous: for though you make that Scale exactly correspondent to that you protracted by (as you ought to do) the Hills and Dales in the ground truly measured, may make a considerable alteration. fig. 32 fig. 32

It is convenient when you plant your Table, that the Needle hang just over the North-point of the Compass under it in the Box; then may you by the Lines overthwart the Frame of the Table, easily draw two Lines quite through the Plot, cutting one another at right Angles, the one pointing at North and South, and the other at East and West. And if your skill serve you to make the Two and thirty Points of the Compass upon the place where they Intersect, and to draw the Forms of the Houses, Woods, and other remarkable things upon the Demesn, and the course of Brooks and Rivers running through it, it will add to your commendation.

And so it will also, if you take in such parcels of Land bounding it, whether common or peculiar:

culiar to other men, as will make your Plot to look handsomely, like a perfect Square or Oblong. But however that be, you must be sure to protract truly all Lanes going into it, or through it, and all Closes of other mens mixed with it; and also all considerable Ponds, Ways and Out-lets, with the Names of the Closes and quality of the Ground, whether Meadow, Pasture, Arable, &c.

CHAP. XVI.

Concerning shifting of Paper.

IN such work as that of the last *Chapter*, it may sometimes fall out (through the multitude and largeness of Fields) that one sheet will not hold your whole Plot, in which case you may help your self by shifting Paper (as we call it) thus.

Fig 33. Let A B C D represent our sheet of Paper that covereth the Table, upon which the Plot of the large piece of Land E F G H I K should be drawn; having made my first station at E, and the second at F, I find my paper will not receive the Line F G: but however I draw so far as it will go to the edge of the paper, and planting my Table again at E, proceed in my Circuition the contrary way to K and I, where I find my self again at a loss for my Line I H, but draw it also to the edge of the Paper; Then with

with the Point of my Compasses striking the Line P O, parallel to the edge of the Paper B C, and the Line Q O parallel to D C, and cutting P O in O, I throw aside for a while, covering the Instrument with a new one, which I mark with the figure (2) for my second sheet. *Fig.* 33.

Upon which second sheet (the leading part whereof is represented by the three Lines meeting in the Angular points A and B) I draw P O parallel to A B the leading edge of the paper, and crossing it at right Angles in the point O, by a parallel to B C, *viz.* the Line O R, being of the same distance from B C, that Q O in the former sheet was form D C. Then with a Rule and a sharp Pen-knife I cut off the end of the first sheet at the Line P O, and applying the edge of it to the Line P O, of the second sheet, so as it may touch that Line all along, and the Line Q O of the former, touch the Line O R in the latter, so as to make one Line with it: I draw the Lines P G, being the Remainder of the Line F G, and the Line O H being the remainder of the Line I H, and from their extremities the Line G H. And if the Plot required it, you might proceed on in the second sheet, and annex a third and a fourth, *&c.* as there is occasion. *Fig.* 34.

These sheets may be pieced together with Mouth-Glew or fine Paste, applying the edge of the former (as you did upon the Table) to the Line P O of the latter.

And note here once for all, that when I speak of applying the edge of the paper to a Line, I mean

mean the preciſe edge cut by the Line P O; but when I ſpeak of drawing Lines to the edge of the Paper upon the Table, I hope none will think me ſo abſurd as to mean the edge that is couched under the Frame, but that my meaning is, that the Lines muſt be continued on the paper till they touch the Frame.

CHAP. XVII.

Concerning the Plotting of a Town-Field, where the ſeveral Lands, Butts, or Doles, are very crooked: With a Note concerning Hypothenuſal, or ſloaping Boundaries, common to this and the Fifteenth Chapter.

Fig. 35. SUppoſe A B C D E divided in the manner of a common Field, into ſeven parts or Doles, belonging to ſeven ſeveral men: Firſt Plot the whole as before hath been taught, then meaſuring from A to B upon the Land: ſet one Note down (as you go along) at how many Chains or Links (or both) the Diviſion is between Dole and Dole, and accordingly mark them out by the help of Scale and Compaſſes in the Line A B on the Paper-plot. In the very ſame manner you muſt meaſure and mark out the Lines O C and E D; which being done, take the Paper off the Inſtrument, and laying it before

fore you on a Table, with the ſide A E towards you; the Compaſſes muſt be ſo opened and placed (as by a few tryals they may) that one foot reſting upon the Table, the other may paſs through the points of Diviſion upon all the three Lines, *viz.* A B, O C, and E D, as in this Figure they do. *Fig.* 35.

If the Content of any one or more of theſe Parts, Butts or Doles, be deſired without Plotting; it may eaſily be done without your Plain-Table thus: Take the breadth by your Chain at the head, middle, and lower end, and adding theſe Numbers together, the third part of their Sum is the equated breadth: by which multiplying the length meaſured down the ridge (or middle) the Product gives the Content.

But both in this caſe, and that mentioned in the 15th *Chapter*, the Figure of a Plot may be ſomewhat diſordered, not only by the unevenneſs of the ground within, for which I have given due caution already (that being both the more common and more conſiderable caſe) but alſo by the great declivity of the Ground where the boundary-lines go, either of the whole Plot, or particular parcels. For whereas in Plotting, every Line is preſumed to be Horizontal (or level) that it may paſs from Angle to Angle the ſhorteſt way, and that every part may be duly ſituated, and none thruſt another out of its right place: If it be not level, but falling down towards a Valley, or riſing up Hill, or compounded of both; a Line over ſuch Ground (though true for the meaſure, and for giving up the Content) will be falſe as to the Plot,

Plot, and therefore muſt be reduced to a level, and ſo taken off the Scale, and protracted. For the doing of this there are ſeveral Inſtruments very proper, eſpecially Mr *Rathburne*'s Quadrant upon the head of his Peractor (though it were better to have a Semicircle than a Quadrant ſo placed) and divers others. But ſuppoſing my Country friend to have no other but ſuch as I have already deſcribed, I ſhall ſhew him a plain eaſie way much uſed by practical Surveyors, eſpecially in *Ireland*, as ſome of themſelves have told me, being the very ſame that he may meet with in Mr *Leybourn*'s Book, intituled, *The Compleat Surveyor*; I mean the ſecond way by him diſcovered.

Fig. 36. Suppoſe A B C to be part of a Hill falling within my plot, my Boundary line going crookedly from A to B, following the Surface of the Ground. To find the Horizontal Line (equal to A C;) I cauſe one to ſtand at the point A (the foot of the Hill) and to hold up the end of the Chain to a convenient height, and gently aſcending the Hill, I draw it level and make a mark where it toucheth the Hill, obſerving the number of Links betwixt mine aſſiſtant's hand and that place, where he muſt take his ſecond ſtanding, and hold it up as before, and ſo I draw it out level again till it touch the place where he muſt take his third ſtanding, noting the Links as before, and ſo proceed, till at laſt from his fifth ſtanding I draw the Chain level to the higheſt point within my plot, *viz.* the point B. And now as the pricked Lines of this Figure put together, are evidently equal

to

to the Line A C: So are the Links noted down at every Station, when ſummed up, equal to the Horizontal Line of that part of the Hill. *Fig.* 36.

In the very ſame manner, only inverting the Order, you may find the Horizontal Lines going down hill, where that is moſt convenient: And if there be both Aſcents and Deſcents in one Line betwixt two Angles, the Horizontal Lines of both muſt be found and joyned together in Protraction.

All this concerning Declivities of riſing or falling Ground, is to be underſtood when they are conſiderable, and a very exact plot required: for ſmall ones, eſpecially when much exactneſs is not expected, are not regardable.

CHAP.

CHAP. XVIII.

Concerning Plotting a piece of Ground by the Degrees upon the Frame of the Plain-Table ſeveral ways, and Protracting the ſame.

HItherto I have ſhewed the uſe of the Plain-Table as ſuch, and I think my Directions have been near as plain as the Inſtrument it ſelf: At which ſome quarrel for its over-plainneſs, expoſing the Art to proud ignorant people, who judging the reſt of the Surveyors work to be as eaſie as looking through ſights at a Mark, and drawing lines by a Rule, are apt to undertake to uſe it, or ſlight the skill of ſuch as do. Others ſay, (and that truly) that for vaſt things, ſuch as Foreſts, Chaſes, *&c.* the Circumferentor is more proper: And every one muſt grant, that in wet Weather, either that a Peractor, a Theodolite, or Semicircle, muſt needs be better than the Plain-Table covered with the paper, which cannot endure wet. Hence it is, that ſome Artiſts have to good purpoſe ſhewed how the Box ſcrewed to the Index, and that made to turn on the head of the three-legged Staff, become a Circumferentor. And if theſe thus fixed be turned about upon the Center of the Table, they will (ſay ſome with good reaſon, Mr *Leybourn* for one) perform the work of the Peractor, much better than the Peractor it ſelf. Others ſhew, (as I ſhall

ſhall briefly) that taking the Inſtrument as it is without the charge of further fitting it, or trouble of removing the Box, the Index turned upon the Center will by help of the Degrees on the Frame, perform the work of the Theodolite, to which the Semicircle is near of kin.

And though I might eaſily anſwer all theſe Objections, by ſaying the firſt is frivolous; ſuch fooliſh Arrogance being eaſily contemned or checkt (if worth the while) by putting ſuch conceited fools upon the harder part of the work. The ſecond impertinent to our purpoſe, who deſign not to plot ſuch vaſt parcels of Land: And the third concerning only an extraordinary caſe, and that well provided for otherways, for ſure no man that hath not a Body of the ſame Metal with his Inſtruments, will ordinarily meaſure Land in continual Rain, (a ſudden ſhower may be fenced againſt by a cover:) and if any be ſo eager upon his work, I have ſhewed how it may be done in the former Chapters of this Book, without planting any Inſtrument at all, by Chain, Scale, and Compaſſes alone: Yet I ſhall ſhew how the Plot of a Field may be taken by the Degrees on the Frame not every way that I could imagine, nor that I could tranſcribe, (for that would be tedious) but two ways only, whereof the one is proper for an ordinary Cloſe, where all the Angles may be ſeen from one Station within it, the other fitting any parcel of Land though much larger, whatever be the Figure of it.

For the former take this Example.

Fig. 37. Let A B C D E repreſent the Figure of a Field to be plotted by the Plain-Table in rainy Weather: I put on the Frame without a Paper the graduated ſide upwards, and plant it in ſome convenient place, whence I can ſee all the Angles as at O; then placing the Index upon the Table, ſo as the fiducial edge do at the ſame time go through the Center upon the Table, and the Lines upon the Frame of the Table cutting it perpendicularly at 360, (where the Degrees begin and end) and 180 (the exact half) I turn about the Table upon the Staff-head, till through the Sights (the Side marked with 180 being next mine eye) I ſee the Angle A, and then ſcrew it faſt obſerving where my Needle cutteth, and by back-ſight cauſing a mark to be ſet up in the Line C D at the point F, that the Inſtrument may be kept firm from moving (or be rectified if it be moved) during the Work. And now the Line A O F paſſing upon the Land from the Angle A, directly under the Sights of the Inſtrument to the mark at F, is (as it were) the prime Diameter whence the Degrees of the Angles are to be numbred, and accordingly I mark the Angle A in my Table hereafter to be exemplified with 360 Degrees. But to proceed, turning my Index with the fiducial edge upon the Center, till I ſee the Thread cutting the Mark at B, the ſaid edge cuts upon the Frame at 76 Deg. 15 Min. which I note down for that Angle:

gle: The like work I do, turning the Sights to C D and D, (but not to F, for there is no Angle, but only a Mark in the Boundary) and I find mine Index to cut for every Angle as I have marked them with n the pricked Circle of the laſt Figure, *viz.* 157 Deg. 35 Min. for C, 225 Deg. 20 Min. for D, and 278 Deg. and 50 Min. for E. *Fig.* 37.

Then I meaſure (or cauſe to be ſtrictly meaſured by others) the Diſtances betwixt the Place where the Inſtrument ſtands, and every Angle, and find them to be as I have ſet them upon the pricked lines in the little Circle, *viz.* A 4 Chains 20 Links, B 4 Chains 3 Links, C 3 Ch 84 Li. D 5 Ch. 35 Li. E 5 Ch. 6 L. And now my Table both for Lines and Adgles is thus perfected, and the Work is ready for Protraction within Doors.

	D	M	C	L
A	360	00	4	20
B	76	15	4	03
C	157	35	3	84
D	225	20	5	35
E	278	50	6	06

Your judgment will eaſily inform you, that in ſuch weather we ſhall hardly ſtand to make our Table neat and formal, but any thing (how rude ſoever) that we can underſtand, doth the feat. A Welſh Slate with a ſharp Style, (or for want thereof, a Black-lead Pen and a ſmooth end of an hard Board like a Trencher) is more convenient at ſuch a ſeaſon than Pen, Ink and Paper. But of all I would commend for expedition a Read-lead Pen, whereby you may mark out every Angle neatly with one touch upon the Table it ſelf, juſt where it toucheth the Frame by help of the fiducial edge, and cloſe by it the

Fig. 37. length of the Line from the Center to that Angle: All which may be eaſily cleared off by a wet Sponge or Cloath ſo ſoon as you have protracted. Or if through the ſpongineſs of the wood, the head of the Table (which we uſe to cover with paper) were made a little reddiſh, what great harm were that? We are forced to do it more real wrong by the points of the Compaſſes in the ordinary way.

Fig. 37. Now to protract our Obſervation: I draw upon a paper the Line A F at adventures, ſo it be long enough, and ſtick a pin in it at pleaſure for the Center O, upon which I place the Center of the Protractor, ſo as the ſtraight ſide (or Diameter) of the Protractor may juſt lie upon the Line A F, the Limb or Arch-ſide being upwards towards B, by help whereof I make a prick or point on the paper at 76 Deg. 15 Min. for B, and at 157 Deg. 35. Min. for C, according to the numbers neareſt to the Limb.

Then turning the Protractor about on the Pin with the Arch or Limb down towards D and E, till the Diameter lie again juſt upon the Line A F, I number downwards from the right hand towards the left, by that rank of Figures that are nearer to the Center, beginning 190, 200, *&c.* and over againſt the places where 225 Deg. 20 Min. and 278 Deg. 50 Min. fall, I prick the Paper at the ſide of the Limb, and through thoſe four points I draw ſo many ſeveral Lines, (having laid aſide the Protractor) upon which and alſo upon the Line A O, I mark out by Points the true meaſure of every Line (by a Scale) from the Center, and from thoſe

thoſe points drawing the Lines A B, B C, C D, D E, and E A, I have the true Plot of the Field.

Where note by the way, that we eſtimate Minutes as well as we can both upon the Frame of the Plain-Table, and the Protractor, accounting half a Degree 30 Minutes; a third part 20; a fourth part 15, *&c.* And though by this means it is impoſſible to avoid ſmall errors, 'tis eaſie to avoid ſenſible ones; and the like may be ſaid when we protract by a Line of Chords, of which I now come to treat.

Having proceeded in the Field as before, and made my Table for Lines and Angles, or done that which is equivalent by a Red-lead Pen, I draw the Line A F, and having extended my Compaſſes to the Radius (or 60 Degrees) on a Line of Chords, I ſet one Foot towards the middle of the Line A F, and with the other I deſcribe a Circle like that in this Figure of a five-Angled Field, but much larger, according to the length of the Radius: Then extending the Compaſſes from the beginning of the Line to 76 Deg. 15 Min. I ſet one Foot in the Interſection of the Circle by the Line A, and with the other Foot make a mark in the Circumference of the Circle upwards towards the right-hand, and through it draw the dry Line B O.

In the next place I ſubſtract the angle 76. 15 from 157. 35, where the Index cut for the Angle C, and there reſteth 81 Deg. 20 Min. which I take off the Line as before, and ſet it upon the Circumference from the Interſection by B O, toward the end of the Diameter marked

Fig. 37. with F, and through the Point were it falleth, draw the dry line C O.

In like manner I ſubſtract 157 Deg. 35 Min. from 225 Deg. 20 Min. and the difference is 67 Deg. 45 Min. which I ſet from the Interſection by the Line C O downwards paſt the prime Diameter A F, and through the point where it falleth, draw the Line D O.

Laſtly, Having ſubſtracted 225 Deg. 20 Min. from 278 Deg. 50 Min. there reſteth 53 Deg. 30 Min. which muſt be ſet downward towards the left-hand from the Interſection by D O, and through the point where that falleth, I draw the Line E O. And now when I have ſet the particular Meaſures upon every Line, and drawn the Boundary lines, as I muſt have done if I had uſed a Protactor, the Plot is finiſhed.

But for better aſſurance that I have done my Work well, I take the meaſure of the remaining Angle A O E upon its proper Arch, *viz.* from the Interſection of the Circumference by A F, to the Interſection by E O, and applying it to the Line of Chords, I find it to be 81 Deg. 10 Min. as it ought to be, for it ſhould be the Complement of 276 Deg. 50 Min. to 360, and ſo it is.

	76. 15
And for further ſatisfaction, I ſum up	81. 20
the Degrees and Minutes of all the five	67. 45
Angles, which for plainneſs ſake I have	53. 30
noted in every one of them on the out-	81. 10
ſide of the Circle in the Figure ſo of-	------
ten referred to, and their ſum is 360,	360.00

as it ought to be, and as here is evident.

My

My Reader may now perhaps expect that I teach him how to take a Plot at two or more Stations, when all the Angles cannot be ſeen from one: But becauſe this is ſo eaſie from the grounds already laid, to any that is Ingenious, and in part rendred unneceſſary by the Method preſently following, I ſhall content my ſelf to give this general hint.

When you have from one Station taken in all the Angles you can ſee from thence, and then are to remove to your ſecond Station, do juſt as you would do if the Table were covered with a Paper; only it is at your choice, whether you would guide your ſelf for back-ſight by a Line that may be rubbed off, drawn upon the Table it ſelf from the Center to the Degrees on the Frame along the fiducial edge, or by noting only what Degrees it cuts on either ſide of the Center, the edge paſſing through it, that by the help thereof and the Needle, the Inſtrument may be placed in the ſame Line and Situation as before, for taking in the reſt of the Angles, if it can be, if not, another Station muſt be taken after the ſame manner. But now to my ſecond Method.

Let A B C D E be the Figure of a Field to be plotted, the Weather being bad: I ſend mine Aſſiſtants to find the length of every ſide, meaſuring it about, *cum ſole*, beginning at A, who return me ſuch an account of every ſide, in Chains and Links, as I have noted them upon the Figure and in the Table following, *viz.* A B 3 Chains, 73 Links, B C 4 Chains, 91 Links, *&c.* In the mean ſeaſon, I make haſte to find the Angles, *Fig.* 38.

gles, and without curioſity plant the Inſtrument at B, and laying the Index on the Center I look at C, and find the Index cutting 10 Deg. 15 Min. and looking at A, it cuts 126 Deg. 45 Min. out of which if I ſubſtract 10 Deg. 15 Min. there reſteth 116 Deg. 30 Min. for the Angle A: but becauſe I like not my Quarters ſo well as to ſubſtract there, I ſet them down thus:

B $\begin{matrix} A \\ C \end{matrix}$ 126. 45 / 10. 15

the meaning whereof is, that B notes the Angle, and C A the Lines meeting there, cutting ſuch Degrees on the Frame; and the reaſon why I ſet A above, is for more ready ſuſtracting afterwards: then removing to the Angle C, and thence looking at B and D, I find the Index to cut as here expreſſed,

C $\begin{matrix} B \\ D \end{matrix}$ 153. 10 / 15. 40.

In like manner I find at D thus, D $\begin{matrix} C \\ E \end{matrix}$ 96. 05 / 28. 50

At E thus, E $\begin{matrix} D \\ A \end{matrix}$ 141. 20 / 11. 45.

And laſtly, at A I find them thus, A $\begin{matrix} E \\ B. \end{matrix}$ 98. 39 / 9. 20

An.	D.	M	Side	Ch	L.
A	89	10	A B	3	73
B	116	30	B C	1	91
C	137	30	C D	4	88
D	67	15	D E	9	64
E	129	35	E A	2	29

This being done, I haſt under Covert, and by Subſtraction find 116 Deg. 30 M. for the Angle B 137 Deg. 30 M. for C. 67 Deg. 15 Min. for D. 129 Deg. 35 Min. for E, and 89 Deg. 10 Min. for A, as you find them

them on the Figure, and in this Table together with the length of the Lines. *Fig.* 38.

Note, that there is a way to find the Angles without Subſtraction, if at every Station you lay the fiducial edge over the Center, and the Diviſions 180 and 360, turning about the head of the Inſtrument upon the Staff, till through the Sights you ſee one of the Neighbouring Angles, for the Index turned upon the Center to the other Angle, will give you the quantity of the Angle you are at, but this exact planting at every Angle is more tedious than the other, and therefore not ſo fit for wet weather. But now to protract this Plot:

Firſt, by my Scale, Rule, and Compaſs, I draw the Line A B in length 3 Chains, 73 Links, ending at the point B: then laying the Center of my Protractor upon the Line A B, ſo as the Center of it be upon the Point A, and that end of the Diameter from which the Numbers are reckoned on the Arch or Limb towards B, I make a point for the Angle A at 89 Deg. 10 Min. by the guidance whereof and the point A, I draw the Line A E, which according to my Scale, muſt be 2 Chains, 29 Links.

In like manner placing the Diameter upon A E, juſt as it was upon A B, and the Center upon the point E, I mark out by the Limb (for the Angle E) 129 Deg. 20 Min. by which I draw the Line E D, 6 Chains, 64 Links.

In the next place, I bring the Center of the Protractor to the point D its Diameter, lying on the Line E D, and its Limb towards A, by which I prick out 67 Deg. 15 Min. for the Angle

at

Fig. 38. at D, and draw the Line 1 Chain, 88 Links.

Laſtly, the Center being at C, and the Diameter upon the Line D C, in ſuch manner as before at other Angles, I prick out by the Limb or Arch 137 Deg. 30 Min. and draw the Line C B, for at B my Plot ſhould cloſe, and if rightly done, the Angle at B will be 116 Deg. 30 Min. and the ſide B C 4 Chains 91 Links, which by meaſure I find ſo to be.

But if I plot by a Line of Chords, I am not bound to this Order, but may go from A to B, and ſo round that way if I pleaſe, which I could not ſo well do with a Protractor, without reckoning my Numbers backward, yet it muſt be granted that a Line of Chords neither doth the work ſo quickly nor conveniently, for this is the way.

When I have drawn the Line A B of a right length, I ſet the Compaſſes to the Radius, and placing one Foot of the Compaſſes in the point B, and with the other deſcribe an Arch of a competent length, beginning at that ſide of the Line A B, that is deſigned to be the inward-ſide, and upon this Arch, 116 Deg. 30 Min. muſt be ſet, but becauſe my Line of Chords gives me only 90, I ſet them firſt on from the Line A B, and then take off the remainder 26 Deg. 30 Min. I joyn them to the 90 upon the Arch, making a Point, through which the Line B muſt be drawn of a due length. In the like manner muſt I do at C E, but the Angles at A and D need no ſuch piecing, being capable of being meaſured out by a Line of Chords at once.

Nor

Nor do your Angles only give you trouble in this kind of work, but oft-times your Lines will be found too ſhort to receive the touch of an Arch upon the Radius, eſpecially if the Line of Chords be large and your Scale little, and ſo it may often fall out when you uſe the Protractor upon ſuch ſhort lines as A E and C D of this laſt Figure: In which caſe a rule muſt be applied to them, and they muſt be extended to a due length that the Arches may meet them without the Figure. And if thoſe Extenſions of lines and deſcribing of Arches ſpoil the beauty of your Plot, the matter is not weighty, 'tis ſo eaſie to be retrieved, for if you lay on a clean paper and prick through every Angle, lines drawn between thoſe points will give you the Plot neat and perfect.

One thing more before I cloſe this long Chapter; the Artiſt ſometime loſeth his labour of Protraction through ſome error in the Field, ſo as his Plot will not cloſe: 'tis therefore good to know before we begin that work, whether it will or no; for which purpoſe if we take a Number leſs by two than the number of Angles in the Plot, and thereby multiply 180, that Product being found to be equal to all the Angles, the Plot will cloſe, and ſo it appears by our Plot in this preſent work: the Multiplier being 3, becauſe the Angles are 5, and the Multipier muſt be two leſs than the number of Angles.

	Deg. Min.
180	89 . 10
3	116 . 30
	137 . 30
540	67 . 15
	129 . 35
	540

This kind of tryal is grounded upon two principles of *Euclid* and *Ramus* mentioned in the firſt and third Chapters of this Book, ſhewing that in all plain Triangles, all the Angles taken together, are equal to two right Angles, and that the ſides (conſequently the Angle alſo) of every triangle Figure, are more by two than the Triangles of which it conſiſteth: But I think it not proper to be large in ſuch things, whereof my young Artiſt is like to make but little uſe; for when all is done, I confeſs with Mr *Wing* in his *Art of Surveying*, *Lib. 6. Chap.* 15. that this way of Plotting by the Degrees, is far more troubleſome, tedious, and liable to error, than the other ways upon a ſheet of paper, and therefore not ordinarily to be uſed, but when neceſſity compelleth us.

CHAP.

CHAP. XIX.

Concerning taking in accessible Distances by the Plain-Table, and accessible Altitudes by the Protractor.

THE Substance of what is to be said for the first of these, is gathered from the Instructions given for Plotting a Field, by measuring only the Stationary-distance; but to make the case more plain to an ordinary capacity: Suppose the Line A C to be the unknown breadth of a River, over which a Bridge of Boats is to be laid, and the General (that he may inform himself what store of Boats and Planks is necessary to be brought down) commands me to tell him the true distance from A where he is at present, to C a little Boat-house on the other side the water. *Fig.* 39.

To satisfie his demand, I plant my Table covered with a paper at A, causing one to set me up a mark at B, at a good distance from me, along the bank of the River (the further the better, if distance do not hinder sight:) Then having chosen a point to represent A, and laid the fiducial edge upon it, I direct my sights towards C and B, and strike lines towards them. Which done, I set up a mark at A, and from thence measure to B, (6 Chains, 32 Links) and so plant my Instrument at B, laying the fiducial edge to the line A B, and turning about the head of the

Fig. 39 Instrument upon the Staff, till through the sights I spy the Mark at A, and then screw it fast.

In the last place, I take 6 Chains 32 Links off my Scale, and set it on the Line A *B*, from A to B, and laying the fiducial edge to the point B, from thence direct the sights to C, and draw the Line B C, meeting or cutting the Line A C in C: So shall the space A C measured on the Scale (*viz.* 8 Chains, 29 Links) be the distance desired: and because the Chain is 22 Yards long, if I multiply 8.29 by 22, the Product is 182 Yards and $\frac{38}{100}$ of a Yard, which by reduction is some little more than 13 Inches and $\frac{3}{5}$ of an Inch.

Now to take the height of a Tree, Tower, or Steeple by a Protractor, without any Arithmetical operation, hang a Plumet with a fine Silk Thread at the Center of it, and hold it stedfastly with that end to your Eye, where the Numbers begin, then look straight along the Diameter, as if you were to shoot in a Crossbow without a Sight (still removing backward and forward as there is occasion) till you see the top of the Tree, Tower, or Steeple, and the Thread at the same time fall upon 45 degrees; so shall the distance from your Eye to the Tree, Tower or Steeple, measured in an horizontal or level Line, together with the height of your Eye above the bottom of it, be equal to the height thereof.

If either for convenience of sight, or any other reason, you think good to set the other end of the Diameter to your eye, then the Thread for the tryal aforesaid must fall upon 135 Deg.

instead

inſtead of 45. Other ways of doing this work by this Inſtrument (or a Quadrant) with the help of Trigonometry, and by other Inſtruments, I forbear at preſent (till I write a ſecond Part) conſidering whoſe benefit is here intended.

CHAP. XX.

Of caſting up the Content of Land by a Table.

TO make up the number of my Chapters to an even ſcore, I ſhall add one at the deſire of my worthy Friend Mr *S. L.* before mentioned (to whoſe Experience and Communicativeneſs I acknowledg my ſelf indebted for the notion of meaſuring crooked Land or Doles at the middle and both ends, marking every where how the Diviſions fall, as is mentioned in the beginning of *Chapter* 17.) concerning the uſe of a *Table* borrowed out of the 46th *Chapter* of Mr *Leybourns Compleat Surveyor*, ſecond *Edition*, *Page* 271, which with the uſe take as followeth.

Links	R.	P.
1		
00000	4	00
90000	3	24
80000	3	08
70000	2	32
60000	2	16
50000	2	00
40000	1	24
30000	1	08
20000	0	32
10000	0	16
8750	0	14
8125	0	13
7500	0	12
6875	0	11
6250	0	10
5625	0	09
5000	0	08
4375	0	07
3750	0	06
3125	0	05
2500	0	04
1875	0	03
1250	0	02
625	0	01

This *Table* consists of three *Columns*, the first containing *Links*, the second *Roods* (or *Quarters of Acres*) the third *Perches*: and the use of it is thus:

Suppose a Field to be 7 Chains and 25 Links long, and 5 Chains 50 Links broad, these by multiplication make 398750 (as here is evident) whereof five Figures being cut off towards the right hand, the Figure 3 signifies Acres, and the rest, *viz.* 98750 denote parts, and to reduce them into Roods and Perchces, I first subtract from 98750 the greatest number of Links in my *Table* that can be substracted from it, *viz.* 90000 (and put down for it 3 Roods, 24 Perches which I find over against it in the annexed *Columns*) and the remain being 8750, I look in the *Table*, and find over against it 14 Perches, which by addition makes 3 Roods 38 Perches: So is the whole Content of the Field 3 Acres, 3 Roods, 38 Perches.

7.25		
5.50		
36250		
3625		
3625		
3.98750		
A	R	P
3	3	24
		14
3	3	38

But

But note here, that if the Remainder after the firſt Subſtraction cannot be found in the *Table*, you may take the neareſt to it, ſo the error will be but part of a Perch.

As for Example.

7.35, being the half Perpendicular, and 9. 23 the Baſe, give for their Product 6.78405. The 6 ſignifies Acres and from the reſt 70000 being ſubducted, (to which 2 Rood, 32 Perches anſwer) there reſteth 8405, which becauſe I cannot find in my *Table*, I take the neareſt, which is 8125, to which 13 Perches anſwer: So the whole Content of that Triangular Cloſe is 6 Acres, 3 Roods and 5 ſquare Perches, and a little better.

	A	R	P
	6	2	32
			13
	6	3	05

But *Manum de Tabula*

I am at preſent taking leave of my Countryman, and ſuppoſing he brought with him any competent ſtock of Natural Capacity, and ſo much Arithmetick as enabled him to Add, Subſtract, Multiply and Divide; I dare make him judge, after he hath as faithfully laboured to underſtand me, as I to be underſtood; whether I have not performed what I undertook in my Title or elſewhere.

FINIS.

ADVERTISEMENT.

The Author hereof useth in *Winter* and *Spring* Seasons to board young Gentlemen and others at his Habitation near *Dunham* in *Cheshire*, and to instruct them in these parts of the *Mathematicks*, viz:

Arithmetick.
- Vulgar concerning
  - *Whole Numbers*
  - *Fractions*
  - *Balancing Accounts.*
- Artificial by
  - *Decimals*
  - *Logarithms*
  - *Instruments*
  - *Symbols or Algebra.*

Geometry and therein
- *The Doctrine of plain Triangles.*
- *Measuring Superficials, and particularly Land, by all usual Instruments.*
- *Measuring of Solid Bodies.*
- *Gauging of Casks.*

The *Doctrine* and use of *Globes* and *Spherical Triangles.*
The principles of *Astronomy* and *Navigation.*
The Art of *Dialling*, by *Logarithms*, *Scales* and *Geometrical* Projection, *&c.*

Vivat Rex.

Floreat Regnum.

AN

APPENDIX

CONTAINING

XII. *PROBLEMS*

TOUCHING

Compound Intereſt & Annuities.

BEING

Part of a Letter ſent by the Author to his worthy Friend Mr *John Collins*, Fellow of the *Royal Society*.

Together with

A quick and eaſie Method to contract the Works of Fellowſhip and Alligation alternate.

LONDON,

Printed in the Year, 1702.

XII.

PROBLEMS,

Touching Compound Intereſt and Annuities, expreſſed in Symbols, *to be reſolved by* Logarithms; *firſt preſented in Twelve ſhort Lines, to the Right Honourable the Lord* Delamer; *afterwards Explained by the Inventor* Adam Martindale, *and by his Conſent preſented to the* Royal Society *by Mr* Collins, *and now applied to pertinent Queſtions in a Practical way, to make them more plain and uſeful.*

THeſe *Problems* are diſtinguiſhed into three Ranks, whoſe *Symbols* are thus to be underſtood.

p Principal,
r Rate, *viz.* 1 *l.* with its Rate, } common to all the three Ranks.
t Time,

a Amount or aggregate, proper to the firſt Rank.

ſ Sum of Principal and Arrearages, proper to the ſecond Rank.

d Difference of Principal and Worth, proper to the third Rank.

Their

Their Capitals ſtand for the Logarithms of the Numbers ſignified by the ſmall Symbolical Letters before mentioned.

D. ſignifies *Data*, *Q.* *Quæſita*, *Prob. Problem*, *Reſ. Reſolution.*

The firſt Rank, concerning Compound Intereſt for a ſingle Sum of Money.

1. Prob. *D.p,r,t.* *Q.a?* Reſ. $Rt + P = A$
2. Prob. *D.a,r,t.* *Q.p?* Reſ. $A - Rt = P$
3. Prob. *D.p,a,t.* *Q.r?* Reſ. $\frac{A - P}{t} = R$
4. Prob. *D.p,a,r.* *Q.t?* Reſ. $\frac{A - P}{R} = t$

Examples relating to the Four Problems in order.

Queſt. 1. *What will* 15 l. 10 *s. amount to in* 12 *years at* 6 per Cent *Compound Intereſt?*

Anſw. 31 *l.* 3 *s.* 9 *d.* 1 *q.* as appears by the work.

R. viz. of 1.06 = 0.25306
t = 12

050612
025306

Rt = 0.303672
P 15.5 = 1.190332

A. 31.188 = 1.494004

Queſt.

Queſt. 2. What is 31 *l.* 3 *s.* 9 *d.* 1 *q.* *due twelve years hence worth in ready money, abating after* 6 per Cent. *Compound Intereſt.*

Anſw. 15 *l.* 10 *s.* as here appears.

A 31.182 = 1.494004
R = 0.025306
t 12

050612
025306

Rt = 0.303672
$A - Rt = P$ = 15.5 = 1.190322

Queſt. 3. *At what Rate of Compound Intereſt will* 15 *l.* 10 *s.* *amount to* 31 l. 3 s. 9 d. 2 q. *in twelve years?*

Anſw. At 6 *per Cent.* as here.

A = 1.494004
P = 1.190382

0.303672

12) 0.303672 (0.025306 = 1.06

0

Queſt.

Queſt. 4. *In what time will* 15 l. 10 s. *amount to* 31 l. 3 s. 9 d. 1 q. *at* 6 per Cent. *Compound Intereſt.*

Anſw. In 12 years.

$A = 1.494004$
$P = 1.190332$

$R.025306) 0303672 (12 = t$

The ſecond Rank, touching Annuities in arrear, grounded upon theſe two Axioms.

1. The Annuity and Rate of Intereſt being given, the Principal correſpondent to that Annuity is in effect given alſo, being eaſily found by the Rule of Three, thus;

As the Intereſt of any Principal (*ex gr.* of 1, 10, 100, 1000, *&c.*) *is* to that Principal: *So* the Annuity of Penſion *to* its Principal.

2. The Sum of the Principal and Arrearages of all the payments being found, the Arrearages alone may be obtained by ſubtracting the Principal from that Sum.

1 Prob. $D.p,r,t.$ $Q.s?$ Reſ. $Rt-\frac{1}{2}P=S$
2 Prob. $D.s,r,t.$ $Q.p?$ Reſ. $S-Rt=P$
3 Prob. $D.p,s,t.$ $Q.r?$ Reſ $\frac{S-P}{t}=R$
4 Prob. $D.p,s,r.$ $Q.t?$ Reſ. $\frac{\quad P}{R}=t$

Examples ſuted to the firſt and laſt of theſe Four Problems.

Queſt. 1. *What will* 33 l. per annum *amount to in* 14 *years at* 6 per Cent. *Compound Intereſt?*
Anſw. 693 *l.* 10 *s.* as here.

6 . 100 :: 33
33

300
300

6) 3300 (550
30

30
30

00

$R=0.035306$
$t=14$

0.70124
0.25306

0.354284

N $Rt=$

$Rt = 0.354284$
$P = 2.740363$

$S = 3.094647 = 124315$
5501

69315

Queſt. 2 *In what time will* 33 l. per annum *raiſe a ſtock of* 693 l. 10 s. *at* 6 per cent. *Compound Intereſt?*

Anſw. In 14 years, as here is evident.

6 . 100 :: 33 . 550

$550 + 693.5 = 1243.5 = s.$

$S = 3.094647$
$P = 2.740263$

25306) 0.354285 (14
25306

101224
101224

0

The third Rank of Problems touching Annuities anticipated or bought for a Sum in hand (or that which is equivalent) at compound Intereſt diſcounted, is bottomed upon the former of the two Axioms above mentioned, and this that followeth:

Axiom. If the difference of the Principal and Worth be once found, the Worth is eaſily obtained by ſubtracting that difference from the Principal,

cipal, which is ever greater, being the Worth of the Annuity at that Rate for ever.

1 Prob. $D,p,r,t.\ Q\,d?$ Reſ. $P-Rt=D$
2 Prob. $D,d,r,t.\ Q\,p?$ Reſ. $D+Rt=P$
3 Prob. $D\,p,d,t.\ Q\,r?$ Reſ. $P-D=\frac{R}{t}$
4 Prob. $D\,p,d,t.\ Q\,t?$ Reſ. $P-D=\frac{t}{R}$

Examples fitted to the firſt and fourth of theſe Problems.

Queſt. 1. *What is* 17 l. 10 s. per annum *to continue for* 11 *years, worth in preſent Money, at* 8 per cent. *Compound Intereſt allowed to the Purchaſer?*

Anſw. 124 l. 18 s. 8 d. *as here is ſhewed.*

8 . 100 :: 17.5 ?

17.5

500
700
100

8) 1750.0 (218.75
0

$R=0.033424$
11

033424
033424

$Rt=0.367664$

 $P=2$

$P = 2.339948$
$Rt = 0.367664$
218175
$D = 1.972224 = 931817$
124933

Note, that if instead of subducting *Rt* from *P*, I had turned *Rt* into the Arithmetical Complement 9632335, and added that to the *P* 2.339948, it would have done the same thing in a more convenient manner, (save that it is not so suited to the letter of the Problems) as here is evident, 10 being rejected (as in this case it must ever be) from the *Index*.

0.033424
11
033424
033424
0.367664
9.632335
2.339948
1.972283

Quest. 2. *In what time will* 17 l. 10 s. *pay off a Debt of* 124 l. 18 s. *allowing the Creditor after* 8 per cent. *Compound Interest?*

Answ. In 11 years, thus manifest:

8 . 100

8 . 100 :: 17.5 . 218.75 = *p*
124.933

93.817 = *d*

P = 2.339948
D = 1.972284

R = 33424) 0.367664 (11

o

Any one that underſtands the very Elements of *Algebra*, may contract theſe twelve Problems into three, for the Firſt of any Rank will by Reduction, Application and Tranſpoſition, produce the reſt. Then the work of the ſecond Rank may be performed by the firſt, if one but underſtand, that inſtead of the Penſion, he muſt take the principal correſpondent to it, and work with it till he have found the Amount, from which the principal muſt be ſubducted when the Arrearages of an Annuity are ſought, and ſo proportionably in the reſt of the Problems.

My noble Lord *Delamer*, only noted down for his own uſe $Rt + P = A$, and $P - Rt = D$: by which and a ſmall Canon of Logarithms, he will quickly anſwer any Queſtion of this kind. But for the help of young Mathematicians, I have ſet them forth thus explicitely.

The only inconvenience of any importance that I can yet diſcover in this method, is, that in both Ranks concerning Annuities, the ſecond and third Problems are rather for demonſtra-

tion of the other, and to compleat the Rounds than for any other great uſe, proceeding upon ſuch *Data* as are ſeldom given for finding the penſions and rate, (or if they were, the Work would be rendred uſeleſs) yet we are not left without ſufficient help to find out penſions or annuities by plain and proper *Data*. For in reference to the ſecond Rank, if the Arrearages (or Stock to be raiſed) with the rate of Intereſt and time, be given for finding out ſuch an Annuity, as at ſuch a rate, and in ſo much time, will raiſe a given Stock by its Arrearages; 'tis eaſily found thus:

Suppoſe an Annuity at pleaſure, and by the firſt Problem of the ſecond Rank, find out its Arrearages: Then ſay by the Rule of Three.

As the Arrearages found, *to* the ſuppoſed Annuity: *So* the Arrearages given, *to* the Annuity required.

For clearing whereof, let this be the Example.

Queſt. *What Annuity will at* 6 per cent. *comp. Intereſt, raiſe a Stock of* 693 l. 10 s. *in fourteen years.* *Anſw.* 33 *l. per annum*, as here is manifeſt.

Suppoſe 30 *l.* then the work goeth on thus:

6 . 100 :: 30
30

6) 3000 (500
300

000

$R.$ 0.025306
14

0.101224
0.25306

R = 0.354284
2.698970

S = 3.053254 = 1130146
5001

630146

630.46 . 30 . : 693.5 ?
30

630.46) 20805.0 (32.999
44046

Here we account 32.999 for 33, being ſo much within an inconſiderable trifle.

Likewiſe in relation to the third Rank, if the Worth (or Price) Rate and Time be given, and the Penſion to be purchaſed be required; find the worth of any ſuppoſed Penſion by the firſt Problem of the third Rank, then the proportion ſtands thus:

As the Worth found, *to* the ſuppoſed Annuity: *So* the Worth (or price) given, *to* the Penſion required; as in the example following.

Queſt.

Quest. *What Annuity to continue eleven years will be purchased for* 124 l. 18 s. 8 d. *at* 8 d. per cent. *Comp. Interest.*

Answ. 17 *l.* 10 *s.* as here.

Suppose 12 *l.* Then

8 . 100 :: 12 ?

12

200

100

8) 1200 (150

8

40

40

00

R = 0.033424

11

0.033424

0.33424

R^t = 0.367664

P = 2.176091

R^t = 0.367664

1501

1.808427 = 6413 : 2

851668

85.668

85.668 . 12 : : 124.933 ?

12

249866
124933

85.668) 1499.196 (17.5

60

If the Interval betwixt every two payments be leſs than a year, conſider what part it is; whether $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{12}$ $\frac{1}{13}$ $\frac{1}{52}$ $\frac{1}{365}$, *&c.* And by the Denominator of the Fraction ſignifying that part, divide the Logrithim of 1 *l.* and its rate of Intereſt, (uſually called *the Logarithm of the Rate*) then the abſolute Number anſwering to that Quotient, being made leſs by an Unite, will be a new Diviſor, whereby dividing the given Penſion ſo payable, half, yearly, quarterly, *&c.* the correſpondent Principal will appear in the Quotient, with which you may proceed, as if the Payments were ſo many yearly ones; as this next Example ſhews.

Queſt. 1. *What is a quarterly Rent of* 3 *l.* 15 *s.* *to continue Thirteen Years and one Quarter, worth in ready Money at* 6 per cent. *Compound Intereſt?*

Anſw. 137 *l.* 10 *s.* 1 *d.* as appears here.

R==4) 0.025306
0.006326==1|01467
--1|

01467

.01467)

01467) 3.75000 (235.623

1059

$\frac{1}{4}R = 0.006326$

Number of Quarters 53

0.018978

0.31630

$\frac{1}{4}Rt = 0.335278$

Ar. C. 9 664721

$P = 2.307600$

2551623

2.872321 = 110119

1371504

Whereas I multiplied the Fourth part of the *Log.* of the *Rate*, viz. 0.006326 by 53 (the Number of Quarters) I might as well have multiplied the whole *Log* of the *Rate*, viz. 0.025306, by 13.25, *i. e.* 13 years and 1 quarter, and it would have been full as exact, or rather more, (two places being cut off from the *Logarithm* ſo multiplied towards the right hand:) I ſay rather more exact, becauſe there remained 2 when I quartered the *Log.* of the *Rate*.

Queſt. 2.

Queſt. 2. *What is* 5 *s.* per *Week to continue* 7 *years and three quarters worth in ready Money at* 8 per cent. *Compound Intereſt?*
Anſw. 75 *l.* 17 *s.* 8 *d.* 2 *q.*

52) 0.033424 (0.000642 = .00148
40
.00148) .25000 (168.918
136
R = 0.033424
t = 7.75
0 167120
2 33968
2 33268
Rt = 0 259036.00
Ar. ✣ .9.740960
P = 2.227675
1681918
1.968638 = 931033
751885

Here I multiplied (not by the Number of Weeks) but by the Number of Years and Quarters, cutting off two Cyphers from the Product. And I alſo took the Arithmetical Complement of 0.259036, *viz.* 9.740963, and added it to the Log. of the Principal inſtead of ſubtracting the Log. 0.259036 from it.

Laſtly, As to the Rate of Intereſt, according to the ſenſe of Dr *Newton*, Mr *Dary*, and others,

thers, I ſee not why this Method may not be made uſe of as well as any other. This univerſal Rule being firſt underſtood: *In Queſtions* belonging to the *Second Rank*, the *greater Rate of Intereſt*, the *greater Arrearages*; and the *leſs Rate*, the *leſs Arrearages*. But in *Queſtions* belonging to the *third Rank*, the *greater the Rate of Intereſt*, the *leſs* the *preſent Worth* (or price) and the *leſs* the *Rate of Intereſt*, the *greater* the *Worth*, *& vice verſa*. I ſhall ſhew this in anſwering the Queſtion following, concerning a yearly payment of 1 *l.* becauſe that by bare Multiplication may eaſily be applied to any other.

A Field worth 1 *l. per annum* clearly, is offered to be let 11 years for 15 *l.* to be paid at the end of thoſe years: What is the rate of Intereſt demanded for the ſeveral payments?

Anſw. Firſt, I ſuppoſe 6 *per cent.* and trying it by the firſt Problem of the ſecond Rank, I find the Arrearages to be but 14*l.* 19*s.* 5*d.* which is 6 *d.* 3 *q.* too little.

Then trying what the Arrearages will amount to at 6.2 (or 6*l.* 4*s.*) *per cent.* I find 15. 13, or 15 *l.* 2 *s.* 7 *d.* 1 *q.* Perceiving then I have over-ſhot my Mark, I make my third tryal at 6.1 (or 6*l.* 2*s.*) *per cent.* and the Reſult is 15.05, or 15 *l.* 1 *s.* which is pretty near, being but 1*s.* above the truth.

But that I may yet bring it nearer, I firſt ſubtract the Reſult at 6 *per cent.* which is 14.971 from the Reſult of the Arrearages at 6. 1. *viz.* 15.05 (becauſe theſe are the Reſults that come neareſt to the truth) and their difference is .079.

2 I ſub-

2. I ſubtract the Rate 6 from the Rate 6. 1, and there reſteth .1.

3. I ſubſtract the neareſt Reſult to 15, *viz.* 14 971, from 15 the given Arrearages, and there difference is, 029. Then working by the Rule of Three, I find (as followeth) .0367

.079 . 1 :: 029

1

.079) .0290 (.0367

7

Which .0367 being added to 6, gives *6 0367*, or *6l. os, 8d. 39. per cent.* Compound Intereſt. And that this Rate exactly fitteth, is plain by the following work.

6,0367 . 100 :: 1

1

6.0367) 100.0000 (16.565

20245

R. 0.025456

11

0.025456

0.25456

0.280016

1.219191

1.499207 = 31|565

16|565

15|1000

But perhaps it will be ſaid, that though in this and all other matters foregoing, I agree well enough with the learned Authors before mentioned, and others that have writ of theſe Subjects; yet in my Country Almanack for the Year 1677, the Fourth *Problem* agrees neither with them, nor what I have here written, but apparently claſheth with them.

I confeſs this charge had been very juſt had that *Problem* been deſigned for the ſame end, and produced a *different* effect: (for in ſuch a caſe to differ is to contradict.) But foraſmuch as in my very entrance upon it, I began thus. *There is a* Problem *in ſome learned mens Works, ſeemingly of the ſame importance with this, but indeed much different both in the Deſign and Effect*: And after in anſwering Objections ſhow wherein the Deſign differs; I cannot but admire the Undertakings of that able Artiſt, who was ſo much at leiſure as to prove with a great deal of Pomp, that mine will not attain the end for which I declared it was never deſigned, as clearly as words can utter it. A Sword may be a very good one, yet a very bad Inſtrument to fell Trees with But I hear he is dead, and I ſhall rather lament the loſs of him, and divers other famous Mathematicians (which Death hath of late deprived us of) than unneceſſarily to expoſe the impertinence of his Paper to the Publick View; or ſo much as name him to the prejudice of his Memory. But from henceforth I expect from all ingenious perſons, that they neither take for granted what I profeſſedly deny and diſprove; nor urge mine Objections againſt my ſelf,

ſelf, without taking notice of mine Anſwers.

For underſtanding of what follows, it were very convenient to have inſerted herein the whole Diſcourſe in the Country-Almanack about this buſineſs; but becauſe this little Paper cannot afford it room, I ſhall only point briefly at the Deſign and Management; and anſwering all the Objections I ever met with of any ſeeming Importance, make things as clear as I can in ſo narrow a compaſs. The deſign was to diſcuſs in plain language (ſuitable to Country-mens underſtanding) this practical Queſtion, wherein they are oft concerned; *viz.* Whether it be more advantage for the Leader to receive for 100 *l.* in hand, Compound Intereſt at 8 *per cent. viz.* 71 *l.* 7 *s.* 7 *d.* 3 *q.* at the end of ſeven years above the ſtock of 100 *l.* ſuppoſing it can be legally aſſured (as in *Ireland* it may, and in *England* by equivalence in Goods or Lands, without mentioning Intereſt) Or to enjoy a Farm for ſeven years in conſideration of 100 *l.* that will clear him juſt 20 *l. per annum*?

My Anſwer was, that the former was the better Bargain (and conſequently the higher Rate of Intereſt for the whole Stock during the whole term,) for which I gave in theſe grounds under the feigned perſons of *A.* and *B.*

1. *A* is to receive (as is ſaid) 171 *l.* 7 *s.* 7 *d.* 3 *q* and if *B*, who takes the Farm, receive not equivalent, his rate of Intereſt is lower.

2. *B* receives only 140 *l* by 20 *l. per annum*; only he hath the advantage to improve the ſeveral payments from the time they grow due, to the end of the term

3. Theſe muſt be computed at ſome certain rate of Compound Intereſt: For to Compute them at no rate of Intereſts, or at Simple Intereſt, or one taken up arbitrarily, will not ſute the caſe.

4. The rate tolerated by the Statute, *viz.* 6 *per cent.* under which none will take, and above which none dare expreſly bind any to give, and at which any reſponſible man may be fitted, is to be preferred before any other.

5. At this Rate the 7 payments will amount but to 167 *l.* 17 *s.* 6 *d. q.*

Of theſe 5 Pillars, the 1, 2, and 5th, were never attempted to be ſhaken by any that I know of: What aſſaults have been made againſt the other two, or deſign of the whole Fabrick, I ſhall briefly conſider; and I find them (beſides thoſe anſwered when I firſt publiſhed that Problem) to be theſe three:

1 *Ob.* The Queſtion is not at all, how the Receiver improves his Payments, but what Improvements was made in the Debtors hands, and the ſame Rate carried on?

Anſw. We agreed that the Rate is not to be computed according to what the Uſurer actually makes of it. If he gives away, loſes, or lends freely all the ſeveral Payments: Or if he make new Bargains more oppreſſive than the Original one; the true value of the loan of ſo much Money for ſuch terms, is one and the ſame But I utterly deny that it is to be reckoned, as if the ſame Rate muſt be carried on, and that for unanſwerable Reaſons (as I ſuppoſe them to be) laid down in my former diſcourſe, or eaſily colligible from it. 1. The

1. The Laws of our Nation prohibit upon ſevere Penalties, the taking of more than 6 *per cent*. therefore that is the utmoſt of the legal worth, which he that exceeds, runs (as I take it) the hazard of all, and a great fine.

2. If he can evade the penalty, no reſponſible man needs to give more than 6 *per cent.* and few are willing to give more than they need.

3. If he get over both theſe blocks, and make new oppreſſive Bargains, this is nothing to the Queſtion as I propounded it; for that was, whether *A.* or *B.* received the higheſt Intereſt for his Money, *by vertue of the above mentioned Bargains*, not by vertue of occaſional after Bargains. When the Debtor hath paid in a years Penſion, he hath done with it; and if he have it not ready, he may take it up at ordinary Intereſt, and the proportion is broken off for ſo much.

2. *Object.* If we regard Laws in the caſe, what need we any Rules concerning Compound Intereſt, ſeeing the Laws of the Land allow only ſimple Intereſt.

Anſ. 1. The Laws are not againſt Compound Intereſt, as it may be managed: that is, the Uſurer may receive his ſimple Intereſt at the years end, and put out that as a new Stock, and ſo undeterminately from time to time; or if you will call this ſimple Intereſt, it's the ſame to the purpoſe.

2. I am told they have ways in *London* for putting out very ſmall Sums to mean Traders upon ſufficient ſecurity; and then it is both legal and practical, though I ſay not how lawful be-

fore God, eſpecially as I hear ſome uſe it.

3. *Object.* It ought to be a Queſtion only of Art, without dependance upon Laws and Uſages: it being the nature of Art in theſe kind of Queſtions, not to ſhew ſo much what ought to be done, as what is really done.

Anſ. 1. It is one Queſtion, what is the nature of Art, and another how far Art is concerned: that is, whether nothing elſe ſave the Rules of Art be regardable in the caſe; which I deny, becauſe Laws and Uſages have a great influence upon it, hindring the continuance of the proportion.

2. I conſider not the Laws as pinching the Uſurers conſcience (as to what is lawful) but as tying up his hands, and ſo obſtructing the proportionable increaſe of his gain.

3. Whatſoever may be ſaid of Art (as Art in the ſtrict notion of it) the Artiſt muſt not be ſuch a Slave to the Rules of it, as not to allow for unavoidable Obſtructions and Irregularities. I hope I may be allowed to tell my Scholars that learn Navigation, that though the direct courſe from one Port to another be upon ſuch a point of the Compaſs; yet other Courſes muſt be ſteered ſometimes in regard of Rocks, Shallows, crooked Channels, Currents, Trade-winds, convenience of Harbour, and freſh Supplies: Or to avoid Pirates, Enemies, Ports, and Places where great Cuſtoms or Payments will be exacted; and many ſuch things which the experienced Seaman is better accquainted with, than I with their Names. And to ſay to ſuch as learn Merchants Accompts, that though it were more Arti-

Artificial and Rational, that Rebates of Intereſt for Money paid before it be due, ſhould be computed at Compound Intereſt (as certainly it is:) yet foraſmuch as it is uſual with Merchants to allow no more than ſimple Intereſt (as appears by printed Accompt-Books) they muſt ſubmit to the Laws of the great Tyrant *Cuſtom*.

An *Artiſt* hath not the ſame liberty of ſuppoſition in anſwering Queſtions (eſpecially ſuch as are real and practical) that he hath in propoſing Queſtions, or receiving them from others, when tryal of skill is only or chiefly intended. I may without abſurdity in this latter caſe demand or find the Amount or preſent worth of any ſingle Summ of Money, or yearly Penſion for any terms propounded at any rate of Intereſt given, though ſuch as is never likely to come in practice: (*ex. gr.* at 2, 3, 14, or 20 *per cent.*) But if I be put by the nature of my Work, to compute the Amount or preſent worth of Payments, no rate of Intereſt being named, I ought prudently to weigh all Circumſtances, and pitch upon that which is poſſible and rational. And this is not a work of *Art*, but of *diſcreet Judgment*, (wherein great reſpect is to be had to penal Laws, Uſages, and other obſtructions) after which *Art* takes it own *Province*, in computing after ſuch a Rate reſolved upon.

To make all this plainer (if poſſible) than I made it at the firſt Publication; Let us for once ſuppoſe a rare caſe, *viz.* That *B* a *Mathematician* turns Uſurer, and for 100 *l.* ready Money, takes a Farm for 7 years, that he lets to another for 20 *l. per annum* clearly, knowing before

fore hand he could ſo let it: and *A* his familiar Friend thus accoſts him; *I wonder to hear you are grown ſuch an Extortioner, as to receive* 20 l. per annum, *ſeven years together for* 100 l. *B* anſwers, Before you find fault with the Mote in mine Eye, take the Beam out of your own: You have bought the *Reverſion* of a piece of Land after ſeven years for 100 *l.* for which I will *bona fide* give you 172 *l.* at the time of your Entrance upon it; which I find by the Rules of mine Art to be 12 *s.* 4 *d.* 1 *q.* above Compound Intereſt at 8 *per Cent.*

A replies, Learned men, ſay ſuch a Bargain as yours will clear 9 *l.* 3 *s.* 4 *d. per cent.* and better. *B* rejoyns, 'Tis true they do ſo, but then they ſuppoſe the ſame Rate to be continued, (for none will ſay, that if the Payments lie by unimproved, or be let out at an under Rate, that in ſuch caſes that Rate of Intereſt can be anſwered) and this ſuppoſition is really impracticable; or (to ſay the leaſt) that which a rational man cannot depend upon. I did accidentally meet with this Bargain from the hands of a weak man, which yet I durſt not have accepted (the Laws are ſo ſtrict, if it had been a Rent charge in Money. And will you undertake to find me Fools that ſhall at every years end, take off the ſeveral Payments, yea and all the increaſe of them as they grow due, upon the ſame terms that I put out the 100 *l.* upon, and to let me Land for it? If you think you can, I will make you a great Bargain, (becauſe I know you to be a punctual reſponſible man) you ſhall receive the firſt yearly payment,

and

and at the end of the ſecond year, pay me ſimple Intereſt for it at *6 per cent.* as the Statute allows, and I will inſtantly return it you, together with the ſecond 20 *l.* which we will joyn into one Sum, and you ſhall take it at the ſame rate: and thus we will do every year to the end of the term, ſtill adding that years growth to what was before, and ſo keep up all the Accounts to *6 per cent.* and if you can make any more profitable uſe of it, much good may it do you, and I will heartily thank you to boot, for helping me ſo readily to place out my ſmall parcels without loſs. But if you dare not do this, let ingenuity mollifie your charge, and I ſhall not quarrel with you about your rate of 9 *l.* 3 *s.* 4*d.* which perhaps you borrowed (or ſome one for you:) from Dr *Newton*, his *Trigonometria Brtiannica*, *P.* 37. for I acknowledge it is that and ſomewhat more, though not much; but I ſay (which I deſire you to obſerve) that I receive no compound Intereſt after that rate, but only ſimple: and that not for the whole Sum during the whole term (as you do compound Intereſt for all your Stock) but only for the whole 100*l.* for the firſt year: for you know that if at the end of that year 20 *l.* be ſubtracted from 109 *l.* 3 *s.* 4 *d.* all the Intereſt is paid, and ſo much of the Stock as brings it down to 89 *l* 3 *s.* 4 *d.* This 89 *l.* 3 *s.* 4 *d.* being computed at the ſame rate of Intereſt, 20 *l.* abated from it, clears the Intereſt again, and brings down the Stock much lower: and ſo year by year the Stock is dwindled away, till at laſt the ſeventh payment (if the Rate were abſolutely exact, as it is near) clears

clears off all Stock and Intereſt. And this is all I receive, ſave only the benefit of the ſeveral yearly payments, which I offer you (or any other ſolvent man) at *6 per cent.* as aforeſaid: At which rate (abating not a Farthing for loſs of time, but ſuppoſing good places ready for every parcel as it becomes due) it will but amount to 167 *l.* 17 *s.* 6 *d. q.* which being the whole Aggregate of Stock and Intereſt, if we ſubſtract from its *Logarithm.* the *Log.* of 100, and divide their difference by 7, as we are directed by Mr *Wingate's* Λογαριθμοτεχνία, *Chap.* 5. *Prob.* 13. the rate of Intereſt will appear to be 7 *l.* 13 *s.* 7 *d.* 2 *q.*

I profeſs freely, Sir, I cannot withſtand B.'s Arguments; but if any other can, I ſhall neither envy his happineſs, nor deſpiſe, (if fairly and ingeniouſly offered) his *Animadverſions*; in that caſe you ſhall command a Sheet from me at any time, in conſideration of any thing ſo objected; but for ſuch injurious and paſſionate diſcourſes as are apter to provoke than convince, I confeſs, 'tis ſuch a piece of drudgery to anſwer them, that (if I could not eaſe my ſelf by contempt) I ſhould think it hard meaſure that my ſilence ſhould be interpreted as their Victory.

Sir, You will pardon this *Prolix Appendix*, when 'tis conſidered that it is not intended for ſuch as your ſelf, but for ſuch to whom nothing can be too plain, by

SIR,

Your Friend and Servant,
Adam Martindale.

A Compendious Method for working out many Concluſions in Arithmetick, *wherein the* Rule of Three *is often repeated; long ſince invented, practiſed and taught by the* Author, *but never till now publiſhed.*

IN all Books of *Arithmetick* that I have had the happineſs to peruſe, the works of both Rules of Fellowſhip and Alligation alternate are performed by the *Rule of Three*, ſo oft to be repeated, as there are particular proportionable parts to be found; which proving intolerable tedious when the partners in Fellowſhip, or the parcels in Alligation are many, cauſed me to think of this following courſe, every way as plain and uſeful, and much more compendious, which to be ſhort is this:

The Rule.

Divide that which is uſually made the ſecond Number in the *Rule of Three*, by that which is as uſually made the firſt and the common Diviſor; That *Quotient* multiplied ſeverally by the reſpective third Numbers, gives the particular reſults. But here it will be neceſſary if you uſe *Decimal Diviſion*

Diviſion (which I judge moſt convenient) that you continue your *Diviſion* till either nothing remain, or you have ſix places of *Decimals* in the *Quotient*, accounting *Cyphers* (if there be any) into the number. And if upon the *Addition* of your Parcels, the Total amount not juſt to the Sum expected, but an Unite ſhort in the Integers, & three more Figures of 9 immediately following, take it for exact, for ſo it is within a matter of nothing.

All this will be plain by three *Examples*, whereof the firſt ſhall be wrought (for plainneſs) both in the old and in this newer way; the other two (for brevity) this latter way only.

1. *Example in Fellowſhip without time.*

A, B, C } Put into the Common Stock. { 15, 24, 33 } The Sum whereof is 72 *l.* and their gain 63 *l.* what is each Mans part?

72

The Reſolution in the old Method is thus.

A 72 . 63 :: 15

15

315

63

72) 945 (13.125

Or 13 *l.* 2*s.* 6 *d.*

Fig: 1.

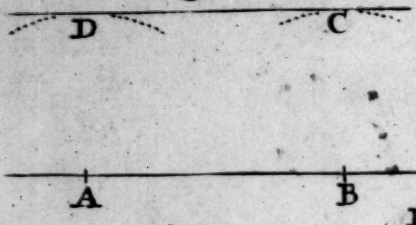
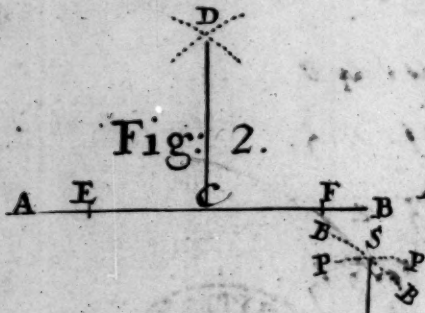


Fig: 2.



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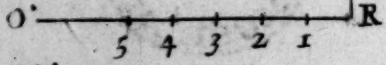
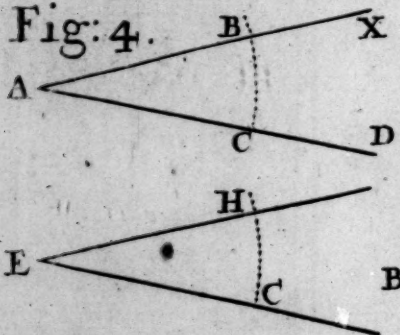


Fig: 4.



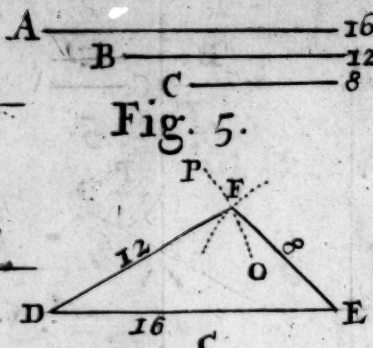


Fig. 5.

Fig: 6.

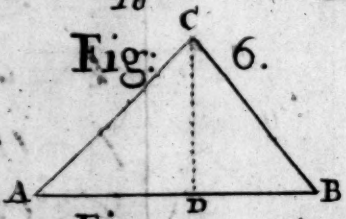


Fig: 7.

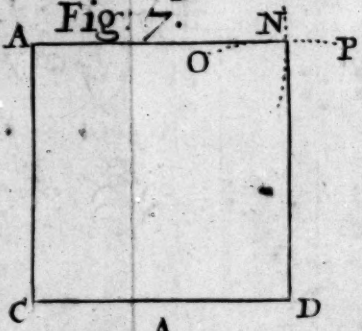


Fig: 8.

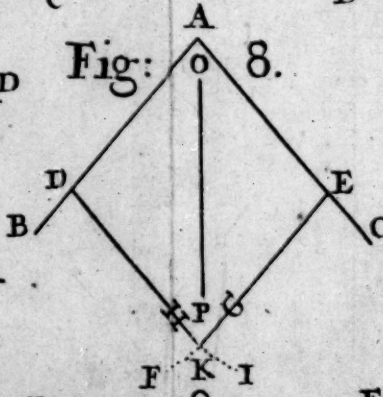
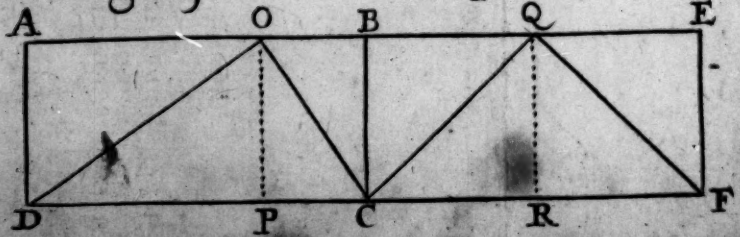


Fig 13.



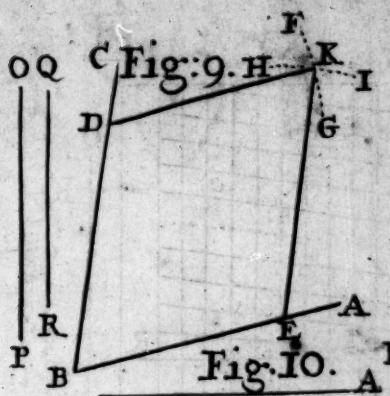


Fig: 9.
O Q
C
F
K
I
H
D
G
A
R
E
P
B

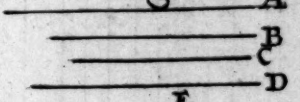
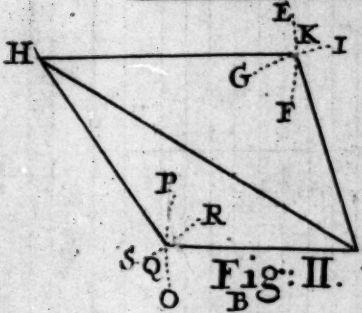
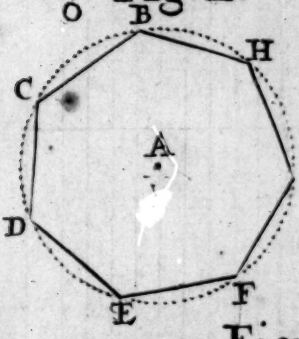


Fig: 10.
A
B
C
D


E
K
I
H
G
F
P
R
S
Q
O
L
Fig: 11.


B
H
C
A
D
E
F

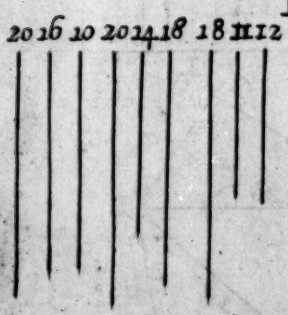

20 16 10 20 14 18 18 11 12

Fig. 12.

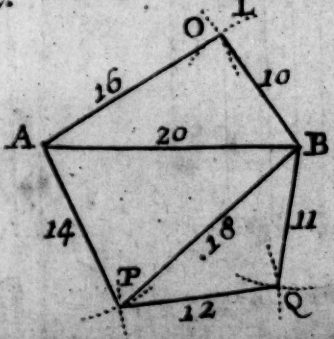
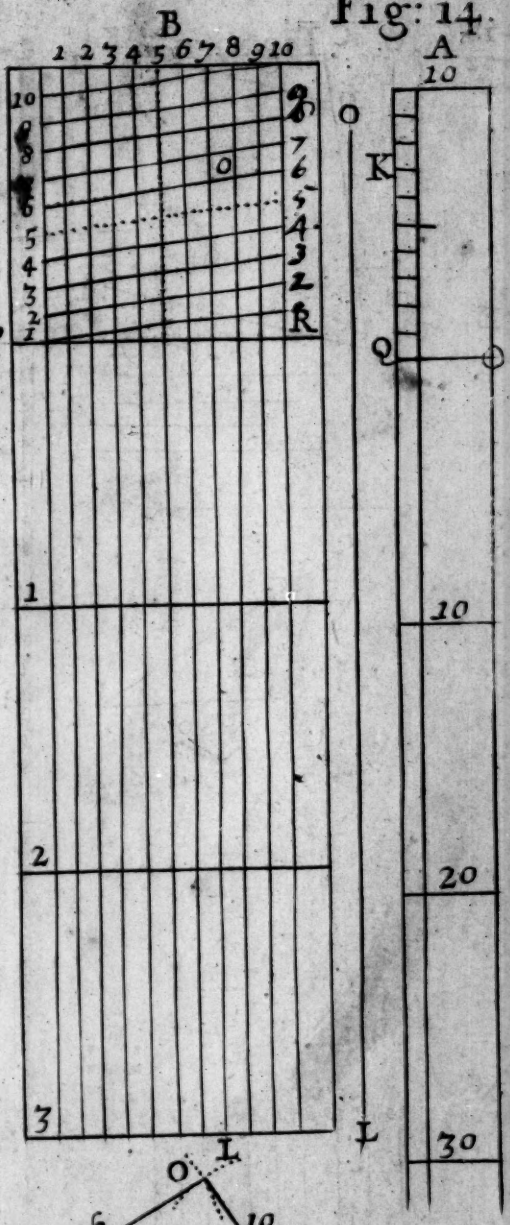

O
L
16
10
A
20
B
14
18
11
P
12
Q

Fig: 14.


B
1 2 3 4 5 6 7 8 9 10
10
9
8
7
6
5
4
3
2
1
O
P
R
1
2
3
O
L
A
10
K
Q
10
20
30

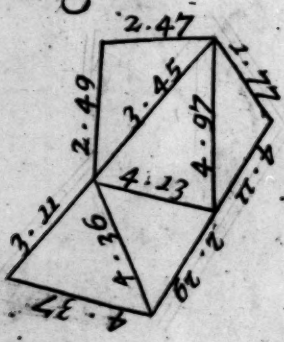
10) 16.5

1	1.65
2	3.30
3	4.95
4	6.60
5	8.25
6	9.90
7	11.55
8	13.20
9	14.85
10	16.50

BRITISH
10 JY 60
MUSEUM

16.5
4.5
10

Fig: 18



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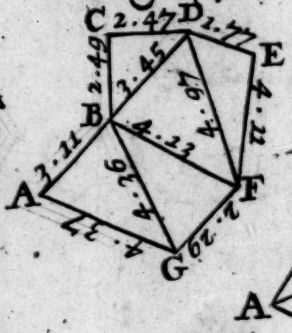


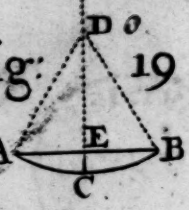
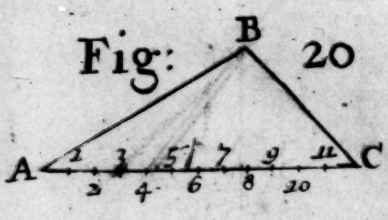


Fig: 19

Fig: 20



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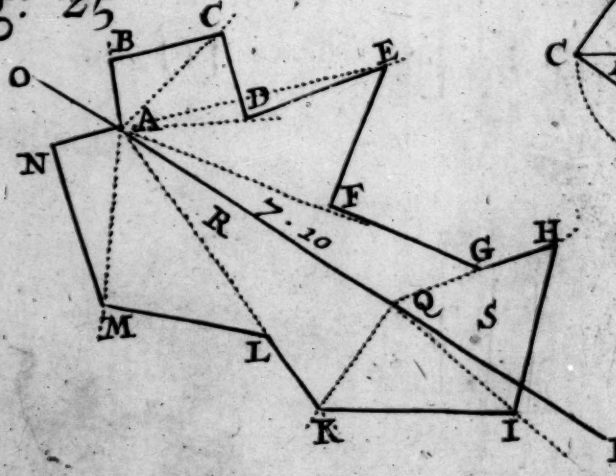


17

Fig: 16

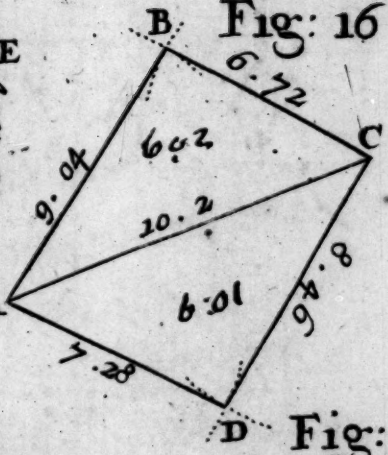
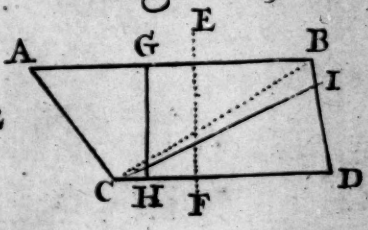


Fig: 15

Fig: 23



20

Fig: 22

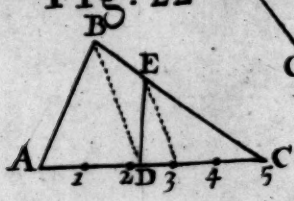


Fig: 21

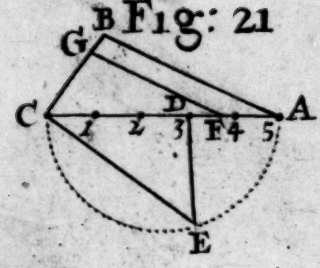
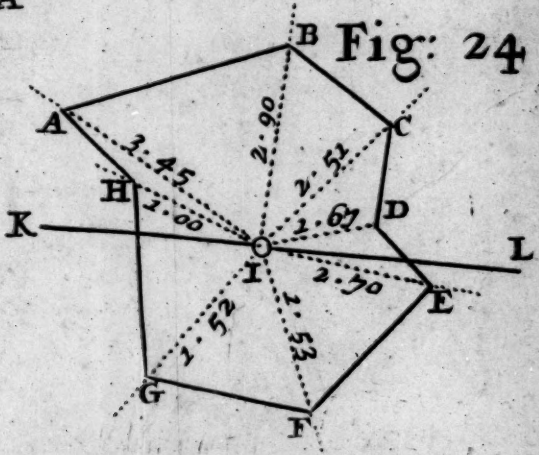


Fig: 24



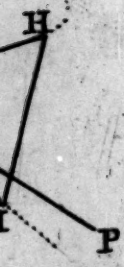


Fig: 26

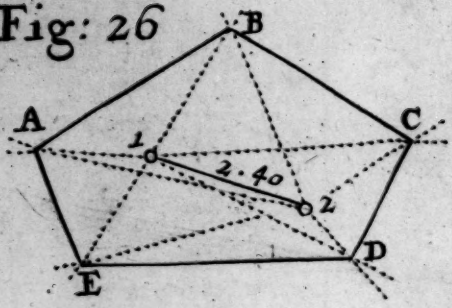


Fig: 27

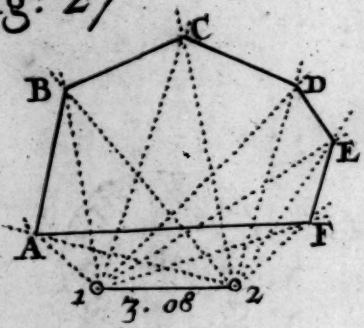


Fig: 28

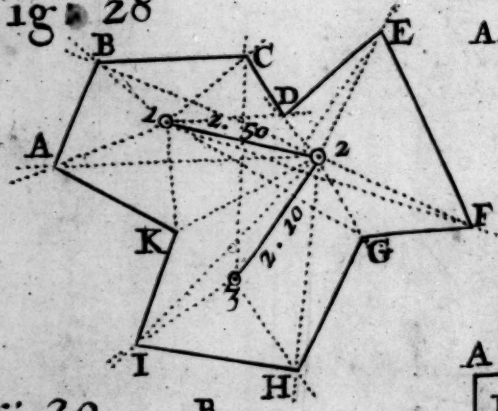


Fig: 29

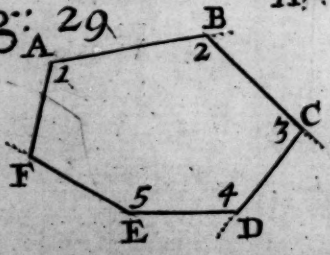


Fig: 30

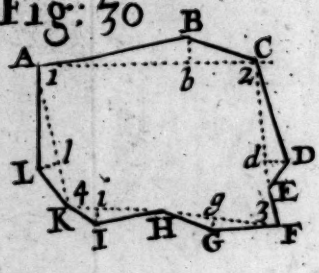


Fig: 31

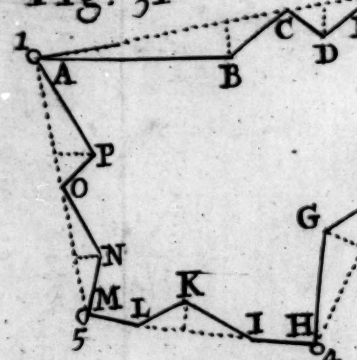


Fig: 32

Fig: 33

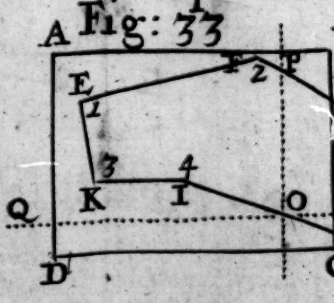


Fig: 34

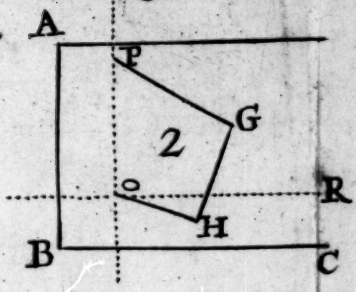


Fig: 37

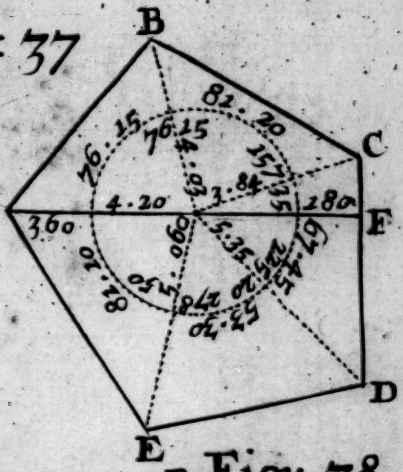


Fig: 35

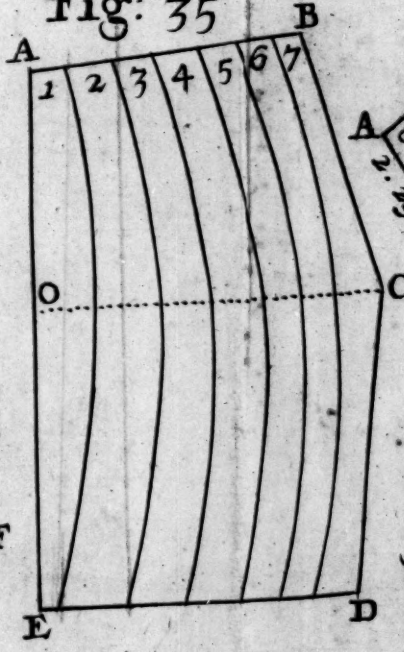
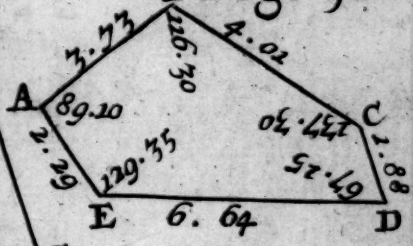


Fig: 38



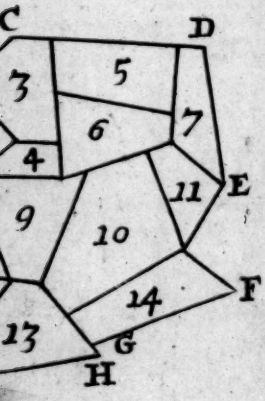
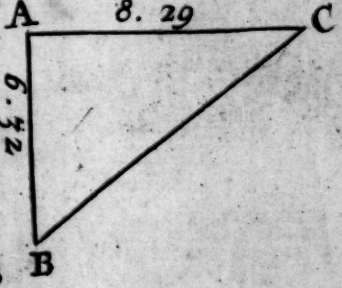


Fig: 39



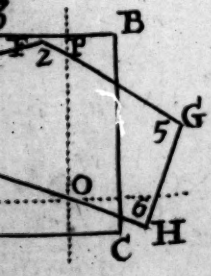


Fig: 36

B 72 . 63 :: 24

24

252
126

72) 1512 (21 *l.*

0

C 72 . 63 :: 33

33

189
189

72) 2079 (28.875
0 Or 28*l.* 17*s.* 6*d.*

13.125 13-02-06
21. 21-00-00
28.875 Or thus, 28-17-06
63.000 60-00-00

In the new way the work is thus done.

72) 63.0 (.875 *A.*875
0 15

4375
875

13.125

B .875	C .875
24	33
3500	2625
1750	2625
21.000	28.875

I hope by this time mine intelligent *Reader* is aware, that if the Partners had been many (as in Voyages and Adventures it oft falls out) the difference betwixt the two Methods would have been yet more signally conspicuous; for one single Division sufficeth, be the Partners never so many: Though I confess there is yet a nearer way by the help of *Logarithms*; of which I shall present this Specimen.

63 = 1.79934055	.875 = 1.94201805
72 = 1.85732250	15 = 1.17609126
.875 = 1.94201805	13.125 = 1.11810931

.875 = 1.94201805	.875 = 1.94201805
24 = 1.38021124	33 = 1.51851394
21 = 1.32222929	28.875 = 1.46053199

The second Example in Fellowship with time.

A, B, C laid { 43, 52, 63 } for { 8, 5, 7 } Months { They gained 73 *l.* 2 each Mans part ?

The

The Resolution by this new Method is thus.

A 43	*B* 52	*C* 63	344
8	5	7	260
			441
344	260	441	1045

1045) 73.00 (.069856
480
A. .069856
344
279424
279424
209568

l. s. d. q.
24.03464 *or* 24--0---7---2

B .069856
260

4191360
139712

18.162560
l. s. d. q.
or 18-3---3---0

C .69856
441

069856
279424
279424

30.806496
l. s. d. q.
or 39--16--1---2

Proof

Proof	or thus			
	l.	*s.*	*d.*	*q.*
24.030464	24	00	7	2
18.162560	18	03	3	0
30.806496	30	16	1	2
72.999520	73	00	0	0

The third Example in Alligation alternate.

Suppose a mixture of Wine of 119 Quarts be required, that must be made up of these several prizes 7*d.* 8*d.* 14*d.* and 15*d.* so as the whole may be afforded at 12 *per* Quart, the parts may be found out in their method (without decimals) thus.

Having linked 8 to 14, and 7 to 15, and Counterchanged their differences from the Common price 12*d*, I find the Sum of their differences to be 14, by which dividing 119 the Quotient is $8\frac{1}{4}$ or $8\frac{1}{2}$ which for convenience of Multiplication we shall change into the improper Fraction, so the Resolution will be thus,

8	}	2
14	}	4
7	}	3
15	}	5

7 ——— 14

Quarts.

$\frac{17}{2} \times 2 = \frac{34}{2} = 17$

$\frac{17}{2} \times 4 = \frac{68}{2} = 34$

$\frac{51}{2} \times 3 = \frac{51}{2} = 25\frac{1}{2}$

$\frac{17}{1} \times 5 = \frac{85}{2} = 42\frac{1}{2}$

119

Having our just Measure of Wine, let us try the Prizes how they suit our purpose by Alligation medial; for considerable errors may be caused by mis-

misapplication of Prices, when the parts were truly taken; but here-under it is apparent that each parcel multiplied by its price, the Sum of the Products is 1428 pence, which divided by 119 give 12 for the common price.

17	34	$25\frac{1}{2}$	$42\frac{1}{2}$
8	14	7	15
136	136	$178\frac{1}{2}$	$217\frac{1}{2}$
	34		42
	476		$637\frac{1}{2}$

136
476
$178\frac{1}{2}$
$637\frac{1}{2}$
1423

119) 1428 (12
119
238
238
0

This Proof by Alligation medial I do not account a needless curiosity; but very useful to be throughly understood, for experience informs me that young men being defective in Skill, Care and Memory, are apt to mistake in several points, but especially one: That is, to esteem the several parcels to be of several prices from which the differences (by which they are found) were originally taken, and not (as the truth is) of the prices to whom in the countrechange they were annexed. As

As to that which seems to look like an unnecessary affection of Novelty, in linking the prices and differences by separate couples; I designed at no higher thing than to free the Printer (if this pass his hand) from the trouble of looking up his dusty Cuts of Semilunes intersecting or enclosing one another, as in our usual Books of *Arithmetick*. Though the truth is, in such cases as this before us, no linking at all is needful; but when the common Quotient is multiplied severally by all the differences, any price above the common may be assigned to any product made by the difference originally belonging to an under price, and contrarily, so as true couples be observed. So here I might have assigned the price 15 to the parcel 34, found out by the difference 4 originally belonging to 8, and the price 14 to the parcel $42\frac{1}{2}$, found by multiplying the common Quotient by 5 the difference of 7 from the common price. But then I must be sure to do justice, so as to assign the price 8 to the parcel $25\frac{1}{2}$ found by 3 the difference of 15 from the common price, and the price 7 to the parcel 17 arising from 2 the difference of 14, as here is plain.

17 x 7 = 119
$25\frac{1}{2}$ x 8 = 204
34 x 15 = 510
$42\frac{1}{2}$ x 14 = 595
1428

119) 1428 (12
119
238
238
0

10 JY60

FINIS.